Mobile-Bed Fluviology

A Regime Theory Treatment of Canals and Rivers for Engineers and Hydrologists

T. BLENCHE
Mobile-Bed Fluviology

A REGIME THEORY TREATMENT OF CANALS AND RIVERS

for

ENGINEERS AND HYDROLOGISTS

T. BLENCHE
D.Sc., F.ASCE, F.Inst.C.E.

Professor of Civil Engineering, The University of Alberta
President, T. Blench & Assocs., Ltd., Consulting
Hydraulic Engineers
Previously Director of Irrigation Research, Govt. of Punjab

THE UNIVERSITY OF ALBERTA PRESS
Edmonton, Alberta, Canada
Copyright © T. Blench, 1969
FOREWORD TO SECOND EDITION

The International Hydrologic Decade, from 1st January, 1965, makes instruction in Hydrology of international importance and urgency and UNESCO (NS/NR/17 or 15th Oct., 1962) lists “evolution of river bed, and sedimentation” as the third of nine major scientific problems of hydrology. The problem has been studied, and become relatively well understood, over several decades by irrigation and other specialized engineers and geologists who, like hydrologists generally, have acquired their knowledge in the exercise of their profession without college instruction. Its literature, therefore, has been disjointed, specialized and non-instructional. The author has attempted to remedy the first and last defects to some extent, in connection with college teaching, by very condensed texts (Blench, 1951 (c), 1957), reference book chapters 1951 (b), 1961, and a very brief illustrated course summary, 1964(b), all based on what has come to be known as the regime theory approach.

The Decade calls for rapid and widespread instruction of students and practising engineers and hydrologists, starting at the novice level and building, step by step, to include the material of previous writings plus recent additions to knowledge. Therefore the present text follows the sequence that has evolved, since 1948, in teaching the two-semester graduate course on quantitative fluviology (entitled River Engineering), in the Civil Engineering Department of The University of Alberta. The first semester serves majors in fluid mechanics, hydrology, soils, structures (with interest in bridges or dams), highways, municipal and sanitary; it uses air photos, diagrams, tables and case histories to give familiarity with river types, phenomena, and principles and shows how the simpler problems of erosion and training works can be solved practically. Chapters 1-5 and light selections from Chapters 6, 7 and 11, 12, 13 should suffice, with the choice from the later chapters dictated partly by questions raised by the class. The second semester is for fluid mechanics majors and others with special interest in rivers; the latter would not read Chapter 14 but, otherwise, all the material of the text would be covered in detail plus some on the mechanics of bed-load transport and suspended load. An undergraduate hydrology course gives instruction on the essence of Chapters 1-5.
Chapter 10 and half of Chapter 12 have been rewritten and provided with extra Figures, Plates and References. They now include recent advances in appreciation of the qualitative and quantitative differences in phase behaviour, particularly those between sand-bed canals and rivers on the one hand and the smaller gravel rivers, laboratory flumes and river models on the other. A special feature of the text is the use of worked problems that have been selected and tested over the years for (a) illustrating principles, (b) applying to the design of canals and river works, and to the analysis of the effects of engineering interferences with river regime. The author hopes that, with some effort to work the problems, the practising engineer will find the book a substitute for college instruction.

T. Blench
The University of Alberta, 1969.
CONTENTS

FOREWORD

CHAPTER 1. INTRODUCTORY
Definition of river, stream, channel and canal 1.1
Sediment 1.2
The basic principle of self-adjustment 1.3
Engineering problems involving the basic principle 1.4
The cost of sedimentation 1.5
Orientation of river study 1.18

CHAPTER 2. RIVER TYPES AND PROBLEMS
Introduction 2.1
Model sand river 2.2
River with noncohesive bed and sides 2.6
Beaver River, Alberta, Canada. Distorted sinusoid 2.7
Pembina River, Alberta, Canada. Serpentine 2.18
North Saskatchewan River, Alberta, Canada. Braided and meandering 2.32
Braiding 2.35
Washita River, Oklahoma, U.S.A. Delta 2.42
Kitimat River, British Columbia, Canada 2.46
Kemano River, British Columbia, Canada 2.49
General 2.50

CHAPTER 3. SYSTEM IN RIVER PARAMETERS
Introduction 3.1
Meanders 3.2
Breadth and depth 3.3
Slope 3.6
Constitution of bed-sand 3.7
Constitution of bed-gravel 3.9
Bed-formation 3.10
An application 3.11

CHAPTER 4. INITIAL TERMINOLOGY AND CONCEPTS
Introduction 4.1
Regime channel 4.2
Regime 4.3
In-regime 4.4
Importance of dynamical statements 7.4
Conditions represented by basic equations 7.5
The basic equations for trifling bed-load charge 7.6
Discussion of the basic equations 7.7
The flow formula adjusted for appreciable bed-load charge 7.8
Status of rough formulas for bed and side factors 7.9
Rough formula between bed-factor and bed-load charge 7.12
Definition of "zero bed-factor", $F_{bo}$ 7.13
Rough relation of zero bed-factor to bed-material 7.14
Systems of measuring particle sizes 7.15
Phase, and indications of dimensional analysis 7.16
Alternative rough formula for $F_{bo}$ 7.17
Practical values of side-factor, $F_s$ 7.18
Derived equations for practical use 7.20
Sectional equations 7.21
Slope equations 7.22
Miscellaneous equations 7.23

CHAPTER 8. FORMULA APPLICATIONS. CANALS OF SMALL BED-LOAD CHARGE OF SAND; DISCHARGE EFFECTIVELY STEADY

Introduction 8.1
Outline behaviour of the real canal system 8.2
Trend of dimensions within a canal system; problem 8.3
Test of certain non-silting non-scouring hypotheses 8.4
Unsuitability of first basic equation for finding $F_b$; problem 8.5
Advantages of third basic equation for finding $F_b$; problem 8.6
Distribution of $F_b$ in a system 8.7
Value of $F_b$ distribution study 8.8
Sediment differentiation in rivers 8.9
Side-factor of a single canal reach 8.10
Mean bed and side factor for a whole system; problem 8.11
Testing for in-regime conditions; problem 8.12
A special lined canal problem; problem 8.13
A semi-regime problem; problem 8.14

CHAPTER 9. FORMULA APPLICATIONS. FLUCTUATING CANAL FLOW WITH MODERATE BED-LOAD CHARGE OF SAND

Introduction 9.1
Equilibrium equations used for non-equilibrium 9.2
Source of bed-load charge 9.5
Problems of assisted river cutoff; two problems 9.6
Rate of bed erosion after enhancing discharge; two problems 9.7
Erosive effect of enhancing slope; two problems 9.8
The effectiveness of Manning's equation; two problems 9.9
Practical use of mean bed-factor for varying stage; problem 9.11

CHAPTER 10. DESIDERATA BEFORE APPLYING FORMULAS TO RIVERS

Introduction 10.1
Definitions related to bed-load transport 10.3
Visible phases in steady bed-load transport 10.4
Simplified phases 10.9
Phases in rivers 10.10
Phase formulas 10.11
The prototype case 10.12
The derivation of formulas 10.14
Phase of basic regime theory 10.15
Graph for $F_{bo}$ 10.17
Subcritical unduned phase of Kellerhals 10.18
Phase of subcritical laboratory flume experiments with $C \to 0$ 10.22
Multiple regression 10.23
Extended formula for subcritical $F_{bo}$ 10.24
Phases connected with $C$ 10.25
Extended application of regime theory slope formula 10.26
Amendment to the $C$ functions of Fig. 7.2 10.27
Reality compared with the prototype case 10.31
Effects of $X \ldots$ factors in gravel rivers 10.32
Effect of suspensions 10.33
Effects of meandering and braiding 10.34
Use of functional forms of steady flow equations 10.35
The meander slope correction factor, $k$. 10.36
Definition of $D$ in gravel rivers 10.37
Surface and grab samples of gravel 10.41
Solution of problems by high-stage $F_{bo}$ 10.42
Samples of gravel for finding $F_{bo}$ 10.43
Rough formula for zero bed-factor, $F_{bo}$, of gravel at large $d/D$ 10.44
Formulas and graphs outside regime theory 10.45
Gravel rivers with negligible charge 10.46
Line or band in Fig. 10.4? 10.49
Comparison of $Q$ formulas with regime theory ones 10.50
Importance of graphed information 10.51
Use of side-factor 10.52
The fourth independent regime equation 10.53

viii
Representative versus equilibrium discharge 10.54
Bed-form measures 10.55
Different viewpoints 10.56

CHAPTER 11. APPLICATIONS OF REGIME FORMULAS TO RIVERS
Introduction 11.1
Test of regime slope formula 11.2
Probability of fit of data to Fig. 3.3 11.3
Mistakes and misunderstandings in data collection 11.5
Data analysis by Regime Slope Analysis Chart 11.6
Analogy with specific speed 11.7
Correlation of S and Q 11.8
Sectional analysis of river regime 11.11
Interference with river regime 11.16
Sectional changes due to changing $F_v, F_n$; two problems 11.17
Special cause of increased river breadth 11.22
Enhanced meander activity 11.23
Design of an assisted cutoff; problem 11.24
Straightening a river 11.25
Estimating bed-load charge; problem 11.26
Sensitivity of transport formulas 11.27
Algebra of calculating $C$; problem 11.28
River factors in flume transport equations 11.29
Obstructions and restrictions 11.30
Economic span for bridge or barrage 11.32
Guide banks for bridges 11.34
Length of guide-bank 11.35
Spurs upstream of bridges and barrages 11.36
Rules for estimating scour 11.37
Flood depth and zero flood depth 11.38
Scour depths for design 11.39
Model information on bridge pier scour 11.40
Flexible aprons 11.41
Apron stone size and thickness 11.42

CHAPTER 12. RIVER MODELS
Introduction 12.1
Small canal as model of large one 12.2
General concept of a model 12.3
Examples of models 12.4
Rivers as models of each other 12.5
Calculation of model scales; problem 12.6
Importance of initial scale calculation 12.7
Analysis of model scales 12.8
Dynamical similarity 12.10
Practical scaling 12.11
The small-scale large model 12.16
Value of qualitative models 12.18
Example of crude scaling for qualitative model 12.21

CHAPTER 13. SEDIMENT EXCLUSION AND EJECTION
Introduction 13.1
Natural differentiation by curvature 13.2
Crump ejector 13.6

CHAPTER 14. PHILOSOPHY
Introduction 14.1
Physical indications of regime formulas 14.2
Potentiality of Blasius Equation 14.3
The “Universal” velocity distribution formula 14.4
The “Universal” flow formula 14.5
Meaning of a “laminar film thickness” 14.7
Postulate of universal flow formula 14.9
Comparison between regime theory development and
boundary layer theory 14.10
Logarithmic flow formulas 14.11
Velocity distributions by dimensional analysis 14.12
To prove a parabolic distribution 14.14
To prove a semi-cubic parabolic distribution 14.15
To prove the vonKarman log-surd distribution 14.16
To prove the Prandtl log distribution 14.17
Meyer-Peter type of boundary movement initiation formula 14.19
Aspects of V^2/d 14.20
Aspects of V^3/b 14.21
Aspects of the Vig Number 14.22
Novel viewpoints on sediment transport mechanics 14.23

REFERENCES
GLOSSARY OF STANDARD TERMS
APPENDIX I. USEFUL DATA
APPENDIX 2. SUMMARY OF FORMULAS
PLATES 1-23
FIGURES 2. 1; 3. 1-4; 4. 1, 2; 5. 1, 2; 7. 1-3; 9. 1; 10. 1-10;
11. 1-4; 12. 1; 13. 1; 14. 1

x
CHAPTER 1

INTRODUCTORY

1.1. Definition of river, stream, channel and canal. Most river reaches with which man interferes or concerns himself have beds of the incohesive materials sand, gravel up to any size, or both. These beds move at some stage of flow. This text is not concerned with channels whose beds are always immobile, so will use the common terms river, stream, channel and canal with the understanding that the beds are incohesive and move at some stage of flow unless the contrary is stated. For historical reasons such channels are often called regime channels; this term will be used occasionally. The side slopes (banks) of a channel are usually of clay, silt, sand, gravel, or a mixture, and are erodible at some high stage of flow but the eroded material seldom, if ever, moves on the sides in the manner it does on the bed; sometimes sides are totally immobile. A canal is an artificial channel.

1.2. Sediment. This word has different meanings in different sciences. Here it means any material, denser than water, that is transported at any stage of the flow. If the term is used without reference to a particular river it means material, heavier than water, that could have been deposited from the water at any time of its career. A one-ton rock is as good sediment as a grain of sand or a particle of clay. The usual constituents of sediment are clay, silt, sand, and gravel which, for the present, may be taken in their popular sense or according to any text on soil mechanics or agricultural soils.

1.3. The basic principle of self-adjustment. The fundamental fact of river science, pure and applied, is that regime channels tend to adjust themselves to average breadths, depths, slopes and meander sizes that depend on (i) the sequence of water discharges imposed on them, (ii) the sequence of sediment discharges acquired by them from catchment erosion, erosion of their own boundaries, or other sources and (iii) the liability of their cohesive banks to erosion or deposition. With this fact appreciated major engineering misjudgements should not arise; when it has been reduced to quantitative terms few major engineering problems will remain beyond useful quantitative solution.
1.4. **Engineering problems involving the basic principle.** The number of phenomena associated with detailed river behaviour, and the variability and even unmeasureability of field factors, often obscure the simplicity of practical applications of the basic principle and the equations expressing them. Therefore a few examples of readily avoidable or foreseeable consequences of engineering interference with rivers are given in the following paragraphs with an outline of the knowledge of simple hydraulics, geology and principles of self-formation that could have permitted their anticipation. The examples have been synthesized from typical occurrences and do not represent any specific happening.

1.5. **Case A.** Province “B”, downstream, persuaded Province “A”, upstream, to agree to its building a dam on a river that flows from “A” to “B”. The dam ponded water right up to the boundary where A had a small town, upstream of which was a fertile alluvial plain. “A” believed that the lake would be an amenity for the town and that the river upstream of the lake would continue to run as usual. Actually, within twenty years the town had to be abandoned because it became waterlogged and subject to enhanced high flood levels, and the enhanced waterlevels could be felt 20 miles upstream by farmers who suffered flooding, waterlogging, and deposition of river detritus on their good land.

1.6. The planners required to know the sediment load of the river, that a delta would form, that a river cannot run horizontally through a delta or anywhere else, that artificially flattening the slope of a self-adjusting river causes deposition of load and that the ultimate fate of the project (if left to itself) must be a reservoir full of sediment and a river running at its natural slope at a height fixed by the height of the dam.

1.7. **Case B.** Country “B”, downstream, persuaded country “A”, upstream, which had rivers “X”, “Y”, “Z” in common to divert “X” into river “Y” for the benefit of irrigation in “B” and, in return, country “A” could have all the water rights of river “Z”. The scheme was worked out entirely in terms of acres of irrigation and kilowatts of power without consideration of what might happen to the rivers. When “X” was diverted into “Y” the latter started to erode its banks and to drop its levels. Within 20 years the breadth of the belt in which the river meandered had increased 50% over some 40 miles of river length, with consequent loss of agricultural land, the river had degraded up to 10 ft. in places, the products of enhanced erosion had filled a reservoir in a canal
system, an irrigation barrage had to be built to meet the rights of riverain dwellers who had been left high and dry by the degrading river, and major repairs had to be made to a dam spillway that had suffered severe undercutting because of the same degradation. 

1.8. Here the happenings could have been foreseen in terms of the fact that the meander sizes, the breadths and the depths must become larger and the slope must become smaller if the discharge becomes larger. The changes are calculable approximately.

1.9. **Case C.** A new highway crossed many meandering rivers. The bridge designers had river crossings surveyed and fixed the foundations of the piers at a certain number of feet below the river bed levels discovered at each pier site on the day of survey. Ten years later many of the high bed levels at piers had become low, and vice versa, many bridge piers and abutments had been propped up by masses of stone revetment and a few had been washed out by floods because stone had not been dumped in time. Many of the approach banks had had to receive expensive stone protection because the rivers had moved over to hit the roads some distance from the bridges and run along them; in one case a flood had gone right over a road and started a new river a mile away from a bridge.

1.10. The basic knowledge required here was that a meandering river must change course cyclically and that the deep channel is at the outside of a bend. The extra depth in bends is roughly calculable.

1.11. **Case D.** Without making measurements of sediment load at all stages of flow a high dam was built to store water for irrigation. After 10 years it was found that the reservoir had lost 15% capacity because of sediment deposition and was therefore no longer a guard against the once-in-20-years drought. All the settlers had occupied the irrigated area under the impression their reservoir would last forever, there was no other site for a large dam, and removal of the sediment mechanically was found to be quite un-economic.

1.12. Suspended load measurements are now routine in most countries. The case reflects the fact that, not long ago, knowledge of the approximate magnitude of a river’s sediment load was not considered important in a storage project.

1.13. **Case E.** Engineers put the large spillway discharge of a reservoir down an old steep creek that had never carried more than a trickle. The original drop over the spillway was 15 ft. After
50 years the drop was 70 ft. and the spillway had been replaced by a chain of drops, each one having been made in the hope that there would be no more retrogression of levels.

1.14. The principle that would have allowed the retrogression to be foreseen is that a channel must adjust its slope to suit the erodibility of the soil. In a sandy soil with no vegetable protection even a couple of cusecs (cubic-ft/sec) will cut its channel down to a slope of the order of 1 foot per mile.

1.15. **Case F.** In a system of unlined irrigation canals (controlled rivers actually) the engineers designed all the big channels to steep slopes so that cut and fill would balance, and all the small distributing ones to flat slopes so as to gain command of the fields. After 10 years they had to protect all the bridge piers on the big channels, which had insisted on cutting down to flatter slopes, and had to face the perpetual cost and nuisance of annual sediment clearance of all the small channels which insisted on depositing their sediment load in an effort to obtain enough slope to carry it along.

1.16. This is from the classic case of the irrigation canals of the Indian continent (Chapter 5). Observation led to the formulas that can be used now to design channels that will be almost in equilibrium as soon as they are run.

1.17. **The cost of sedimentation.** Clearly many ill effects arising from the presence of sediment in rivers can be foreseen and avoided or reduced if present knowledge is spread through the engineering profession. The gain is difficult to express in dollars, but should be an appreciable fraction of an estimated value of the total damage believed to result annually from sediment. A semi-official rough estimate given by Brown, 1948, for the U.S.A. was $150,000,000; the figure should be expected to increase with the number of water projects.

1.18. **Orientation of river study.** To an engineer the costly malfunctioning of rivers he thought he had controlled permanently is, like the malfunctioning of any other work, a spur to investigation of the causes. To a geologist the happenings are interesting manifestations of "instant geology" against which he can test his theories of valley formation. To the hydraulician they are examples of return to equilibrium after an arbitrary displacement, valuable as indicators of the factors involved in the equilibrium. It is logical, therefore, that all three should start to investigate mobile-boundary channel hydraulics from the viewpoint of self-adjustment after a disturbance of equilibrium. This is the approach of
the present text, originating from the slowly accumulated work of irrigation engineers in India and of geologists. Intrinsically there are other logical and scientifically valuable approaches—for example through the mechanics of sediment transport—but none has been so simple or so productive of practically useful results.

1.19. The accepted treatment makes the subject a geo-science so that the “laboratory” is the world and the “experiments” may occupy decades. As a science of this kind must be developed in terms of fully appreciated facts, and the reader can obviously not obtain them rapidly at first hand, the first five chapters will present them at second hand. Obviously they must be placed in an orderly sequence, terminology must be devised, and salient points must receive comment. However, every attempt has been made to keep natural interest and context as if they had been observed directly in the field by the reader; comments are made only when appropriate and technicalities are avoided. Chapters 1-5 are intended to impart sufficient experience and qualitative appreciation of basic principles exhibited by field facts to permit the ensuing theory and applications to be understood and, in any case, to warn against most of the dangers inherent in interferences with river regime. Chapters 6 and 7 lay the quantitative theoretical base needed for analysis and design. The remaining chapters, through Chapter 11, develop approximate practical applications of the basic equations, that are believed dynamically correct for specified simple circumstances, to complex river conditions to which they do not apply rigorously. Chapters 12-14 are rather specialized.

1.20. The whole subject is a relatively new and growing natural science based on its own observations and independent of dogma of related subjects. Therefore the facts, the formulas, and the arguments for practical applications have been stated, perhaps tediously at times, so that the reader can decide their status for himself and, perhaps, amend and extend in due course. He is never asked to accept the common text-book evasion “it can be proved that. . . . .”.
CHAPTER 2

RIVER TYPES AND PROBLEMS

2.1. **Introduction.** The first step in river study is to acquire familiarity with river types and the problems they pose. As years of field experience are needed for first-hand acquisition the present chapter will be devoted to the informal description and discussion of interesting examples which have been chosen from experience to illustrate similarities, contrasts and engineering problems of various types. The narrative is designed to introduce points that will be studied in detail later, and to introduce technical terminology and outlook as they would arise naturally during the acquisition of field experience. More formal terminology will be introduced later. The features discussed are not intended to be exhaustive.

2.2. **Model sand river.** The simplest river, in appearance, has both bed and banks (side-slopes) made of non-cohesive material, does not carry too much of this material, and has fairly constant discharge. Its commonplace demonstration in hydraulics laboratories is usually started as a straight channel cut down the middle of a broad flume ("tray") of clean river-bed sand. The channel never stays straight but, with constant discharge, adjusts itself to have the appearance of a sine curve. The first photo of Plate 1 (a) from Friedkin, 1945, shows such a form after several hours of running; to demonstrate the form the water has been turned down because, at normal supply, it spills over the sand-bars inside the loops so that the incised sinusoidal channel is not visible. Plate 3 illustrates a flood masking the incised channel on the similar river of Plate 2.

2.3. The second photo of Plate 1 (a) shows the same model river several hours later. Notice that the first loop on the left has become more definite at its turning point, the incised channel has moved noticeably downstream near the second turning point on the right, and the channel has apparently jumped to a new course well downstream of the second turning point on the left.

2.4. Conventionally, the channel is said to be tortuous, because it is crooked, and to meander because the tortuosities move. A channel in bare rock can be tortuous but, technically, it does not meander.

2.5. If a river channel switches suddenly, as just illustrated, from
some starting point to a point considerably less than a full meander “wave length” away it is usually said to form a chute, which is one kind of channel switch, or short-circuit, known generally as a cutoff (para 2.18).

2.6. River with noncohesive bed and sides. Plate 1 (b) shows, in a gravel river, a sinusoidal bend resembling a sand model one. Near this bend the conditions are comparable with those of the model. That is, the bed and banks (side-slopes) of the incised channel, and the flood plain, are all of gravel with no cohesive material or vegetation to act as binder. There is indication that a vigorous overspill might occur and cut off the bend nearest the viewer and meet the channel again at a point outside the photo.

2.7. Beaver River, Alberta, Canada. Distorted sinusoid. Plate 2 shows about 5 miles of a sand-bed river of which a reach has been observed and analysed scientifically by Neill, 1965 (a). The photo was taken when the discharge was low so shows the incised channel and represents conditions rather like those of Plate 1 (a) where the discharge had been reduced for the photo. In Plate 3 flood spill hides the incised channel of another reach.

2.8. An incidental interesting peculiarity of the river is that it meanders in a valley cut about 50 feet deep into the prairie and characterised by a beautifully regular but distorted sinusoidal shape in plan. The Canadian prairies were under a continental ice sheet of the order of a mile thick some 10,000 years ago and show several such instances of present-day rivers occupying valleys that appear to be the incised channels of old and far larger rivers. The sizes of the ancient meander bends can be used to estimate the high flood flows of the rivers that caused them.

2.9. The strip over which a river spills during flood is called the flood plain. In the present example the flood plain seems to be the whole remaining valley floor of the ancient river; the flood plain in Plate 1 (a) is visible between scalloped edges and would broaden if the river were run longer; the limits of the flood plain in Plate 1 (b) are extensive. The material of the flood plain is sandy to correspond with the material of the river bed and is fairly uniform but possesses some silt and clay. The plain is covered with scrub.

2.10. The meander pattern of Plate 2 appears to be trying to develop sinusoidal curves against the opposition of the valley walls, whose refusal to yield causes the sinusoids to have flat tops. It appears, too, to have encountered occasional local obstacles that
have impeded its progress and caused pattern distortion. This is not the only indication of progression downstream (i.e. meandering); recent tendency to advance downstream is shown by the incised channel hugging the eroding bank and leaving unvegetated sand bars (white) on the opposite shore; older advances are shown by definite “scars” on the old flood plain, visible more clearly in the inset. Presumably these scars indicate that the whole meander pattern has moved rather spasmodically during each of a series of infrequent large floods; ordinary high flows erode only a little bank each year. There is no conspicuous evidence of a major chute, such as in Plate 1 (a); this may be attributed to the flood plain scrub preventing short-circuiting during flood spill.

2.11. The meander activity is slow in this river despite the clear scar marks on the photos. Air photos are notorious for showing ancient marks as if they were new; even works of archaeological interest may appear relatively recent when seen from the air. Plans of circa 1900 show that the river has deviated only slightly from its position of that time, so some of the air photo scars could be centuries old.

2.12. Meander statistics are interesting and useful despite the unavoidable indefiniteness of quantities to be measured. The meander length is the average wavelength of the pattern, with the wavelength defined as the distance between a point of the river and the next point on the same side of the rough axis of symmetry at which the flow is in the same direction and the pattern has started to repeat itself. For the Beaver the meander length is estimated conveniently as the quotient of (i) the distance along a curved fitting line forming a rough axis of symmetry of a sequence of well developed meanders by (ii) the number of these meanders. This length is about 2,500 feet. The meander breadth is found by constructing a fairly uniform belt embracing the outside edges of well developed meander bends of the incised channel. The average breadth of this belt is the meander breadth, and might be considered as the double amplitude of the wave pattern. A fair estimate is 1,400 ft.

2.13. The meander ratio is the ratio of meander breadth to meander length so is $1,400/2,500 = 0.6$ in this case. The loop chord ratio (LCR) is length along incised channel divided by length along an estimated curved line defining the centre of the meander pattern, so is about 1.3. For some river patterns the estimated curve depends very much on personal preference so the LCR is a relatively poor statistic.
2.14. The flows of the water and sediment phases of the water-sediment complex have not been measured regularly. The water is believed to overtop the incised channel about once in 3 years on an average. During the six cold months a little ground-water seepage flows under the ice and is probably never frozen off completely. High floods can be expected in May from snow melt and low ones in late summer from rain; the worst floods come from severe snow-melt, frozen ground, high slough levels, and rain in combination. Summer water temperature in about 65°F. The sediment that moves along the bed, i.e. bed load, has about 0.5 mm. median diameter by weight (meaning that half the weight of a large sample consists of particles coarser than 0.5 mm. diameter) and has a size distribution typical of river-bed sand (para 3.7). Bed-load movement appears to be negligible till bankfull flow conditions are approached. The movement is in dunes, like those of the desert, with several feet height from trough to crest of the largest ones during very high flood, but only about one foot at bankfull stage; the dunes have been recorded by sonic sounder. Probably the total bed-load flow comes to about one hundred thousandth of the water flow by weight. The sediment that moves in suspension and makes the water appear muddy does not alter flow properties appreciably and probably averages about one ten-thousandth of the water by weight.

2.15. Fig. 2.1 shows a typical cross-section of the valley at a place where the flow was fairly uniform across the incised channel. The bankfull breadth, averaged from various sections, is about 180 feet, with bankfull depth averaging about 9 feet. At bends, during high flood, the greatest depth was at the outside of bends and about 1.6 times the average depth across fairly regular sections. The slope measured along the deep-water channel averaged 0.24 per thousand. The flood plain slopes gently from the incised channel edge downward to the valley wall. This is a common phenomenon of flood plain formation and is demonstrated in canals carrying a mixed sediment load (Fig. 5.1, paras 5.12, 13).

2.16. A basic engineering research problem concerning the depth of scour, below average unbridged river bed, found around bridge piers was one reason for Neill's (1965 (a) ) study of this river. The information given in the preceding paragraphs, even without special technical knowledge, will help appreciation of the several independent factors that he emphasized as important in such scour. First (Plate 3), the highway across the flood plain, together with the abutments, must concentrate more flood flow through the in-
cised portion of the channel spanned by the bridge than when there was no highway; so, as the bed is mobile, one should expect the bed to scour down even were there no piers. Second, if the river changed its course to become curved at a bridge site, one part of the bed might go down to at least 60% more than average depth even were there no bridge or highway (para 2.15). Third, the passage of dunes along the bed (para 2.14) could raise and lower the bed periodically during a flood of constant discharge. Fourth, a pier will disturb the conditions arising from the three preceding factors and produce scour in the drifting sand bed just as a tree produces scour in drifting snow. So the "scour" at a bridge pier has to be studied in terms of river behaviour without the pier, as well as in terms of the effect of the pier by itself.

2.17. Another engineering problem, on larger rivers of almost identical type, is the depth to which to bury high-pressure gas pipelines to ensure safety during high floods. To solve this one must know how deep the bed can drop at the outside of a bend during a high flood, and whether bends migrate rapidly enough to be likely to affect the pipe line on both sides of the river valley during a line's lifetime.

2.18. Pembina River, Alberta, Canada, Serpentine. Plate 4 shows, for low flow, about six valley miles of a river that meanders with snake-like contortions, so might be described as serpentine. The photo was taken during low flow and shows three distinct ox-bow lakes (one dry), which are abandoned and sedimented river bends; they may receive water from river overflow, precipitation, and seepage. Plate 5, from air photos during flood, illustrates, to large scale from Plate 4, the phenomenon of contact cutoff and the outline of many old paths of the river.

2.19. In this reach the incised river channel, like the Beaver, meanders in a shallow prairie valley but, unlike the Beaver, it has succeeded in carving more than enough space for its meander belt; this does not prevent it from contacting the valley walls. The material of the flood plain, which occupies the whole valley, is rather heterogenous. In some places the river cuts banks of almost pure sand, but in others silty and clayey soils are found. Excavations show that the plain is built of apparently nonconforming strata of these materials. The bed has a few feet of sand in some places and is composed of erodible fine-grained cohesive soil in others. The flood plain is cultivated now, but still retains some of the original poplar forest.
2.20. Certain approximate quantitative differences between the Beaver and the Pembina are useful in appreciating possible causes of different meander patterns, see Table 2.1. A peculiarity shown by rows (3) and (6) of the table is that the Beaver floods are relatively high over its banks, and the Pembina floods are relatively low.

Table 2.1. Comparison of Beaver and Pembina Rivers.

<table>
<thead>
<tr>
<th></th>
<th>Beaver</th>
<th>Pembina</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) Flood discharge (cs)</td>
<td>20,000</td>
<td>40,000</td>
</tr>
<tr>
<td>(2) Bankful discharge (cs)</td>
<td>5,000</td>
<td>25,000</td>
</tr>
<tr>
<td>(3) Ratio (1) to (2)</td>
<td>4</td>
<td>1.6</td>
</tr>
<tr>
<td>(4) Flood depth (ft)</td>
<td>20</td>
<td>25</td>
</tr>
<tr>
<td>(5) Bankfull depth (ft)</td>
<td>9</td>
<td>20</td>
</tr>
<tr>
<td>(6) Ratio (4) to (5)</td>
<td>2.2</td>
<td>1.25</td>
</tr>
<tr>
<td>(7) Suspended load</td>
<td>slight</td>
<td>moderate</td>
</tr>
<tr>
<td>(8) Bankfull breadth (ft)</td>
<td>180</td>
<td>250</td>
</tr>
</tbody>
</table>

The suspended load in the Pembina is of quantity and quality that make it deposit on banks (side-slopes) during low flood and cause the channel breadth to contract; the load left on the flood plain after spill is said to contain enough clay to cause a characteristic pattern of cracks on drying.

2.21. With a considerable and mixed suspended load of material ranging from medium sand down to clay the conditions exist for building up a high and heterogeneous flood plain. The fertile fines, unlike the coarser fractions of the sediment load, are found normally in relatively high concentration right up to the water surface. They deposit during overspill on low areas left by recent meandering and induce plant growth that prevents surface erosion and aids the accumulation of further deposits. So, eventually, the flood plain becomes so high that hardly any floods can overtop it; general overtopping seems to occur about once in 10 or more years on an average. The gentle downward slope of the flood plain from river edge to valley wall, observed in the Beaver (para 2.15), occurs here also but at higher relative level.

2.22. With a high flood plain, not only are chutes impossible but also the full force of maximum flood is resisted by banks; with a low flood plain an important part of the flow skips the bends. The exact mechanism of erosion in the two cases of containing and partially containing banks is not fully agreed upon. However, with the general difference appreciated, it is not surprising that, when
there are fully containing banks, sinusoidal meanders grow larger by "inflating" their tops and bottoms till they look like double-S's or a sequence of pairs of complete sine waves turned back to back. The Plates show both inflated and uninflated sinusoidal meanders.

2.23. The final stage of the double-S is that opposite S's meet and a contact cutoff occurs. As the upstream channel at the contact runs higher than the downstream one the cutoff results in a small "waterfall" that projects a high speed jet at right angles to the common tangent at the contact. This jet hits the far bank of the lower channel and starts a new loop eroding into it. Crickmay, 1960, has described Pembina meander development in detail. Plate 6, from his work, shows a waterfall in the Pembina bed shortly after the contact cutoff in the centre of Plate 5 had occurred. The drop is abrupt because the bed was locally of poor sandstone, or consolidated sand.

2.24. The channel upstream of a newly-formed "waterfall" suffers an immediate drawdown of water levels and a subsequent slow degradation of the bed as a consequence; this degradation is accompanied by further drawdown of water level. Downstream, the bed aggrades. Therefore the part of the total flow that continues to follow the cutoff loop suffers a reducing "head" and the consequent reduction in its velocity causes sediment it carries to deposit on the bed and side-slopes. During large floods the sediment that enters and deposits will contain sand; during smaller flows it will be more silty or clayey and these materials will tend to deposit. Thus the old loop "silts up" with layers of material whose constitution depends on the sequence of floods. The heterogeneous nature of the resulting flood plain accounts for some of the contortions of later meanders.

2.25. Plate 5 shows the "scars" of ancient meanders in positions corresponding to major floods of the past and indicates old contact cutoffs also. As any long excavation must cut through such scars and ancient sedimented cutoffs one reason for excavations in flood plains often disclosing nonconforming strata (para 2.19) is obvious.

2.26. Although the double-S, combined with violent cutoffs, may be taken as characteristic of the serpentine meander pattern, distorted bends of apparently different serpentine type appear. Various explanations are possible. For example, a pinched bend rather like a zig-zag in a Chinese cracker firework could have developed from a cutoff such as in Plate 5, and long straight reaches could result
from contact with a relatively resistant stratum that had been deposited from rather still water, or formed part of valley wall.

2.27. The movement of bends that characterises meandering is not the slow sliding of the whole pattern downstream seen in the Beaver. Instead it is a destruction and recreation of bends by contact cutoffs and consequent erosion.

2.28. Meander length in the Pembina, determined from a dozen fairly similar continuous wavelengths, is about 5,000 feet. Meander ratio is not easy to estimate but is about 0.8 determined from the better shaped large double-S bends. Loop chord ratio is about 2.6. Channel slope is about 0.25 ft. per thousand along the flow where the bed is sand or soft clayey earth, and is distinctly more in gravel even after deducting the rapids.

2.29. An interesting feature of the whole river is that, in several miles near the mouth into the Athabasca River, meanders are not obviously different from those of Plate 4 though the pattern is "frozen into" the prairie about 100 feet deep and there is no effective flood plain; there is even an incipient contact cutoff about to isolate a 100 ft. high island in its loop. The bed here is gravel with occasional shale outcrops. Frozen-in meanders appear to be due either to the land rising and the river retaining its natural slope by erosion, or to the river losing sediment load as the effects of deglaciation recede and, therefore, eroding down to a flatter slope.

2.30. A special engineering problem of this river has been how to alleviate occasional flooding. The method chosen was "assisted cutoffs", consisting of small straight channels excavated tangential to the channel across the ends of very long meander loops as in Plate 7. These do not act as violently as natural contact cutoffs, and the condition of tangency ensures that the result will not be rapid development of a new bend in an objectionable position; however they do enlarge themselves fairly rapidly, as intended, to full river size. The engineer has to locate and introduce them according to a long term plan that has allowed for the ensuing long range self-adjustments; unplanned cutoffs can induce erosion that causes more harm than good.

2.31. Subsidiary engineering problems concern the location of roads and bridges so as to reduce protective works to a minimum, and the design of protective works against the unavoidable attacks that do develop. When the nature and limits of meandering are understood and site conditions have been analysed, cheap anticipatory
work can often remove the need for expensive last-ditch defences. The railway bridge in the central loop of Plate 5 was located against a relatively resistant portion of valley wall and has given negligible trouble over half a century.

2.32. **North Saskatchewan River, Alberta, Canada. Braided and meandering.** Plate 8 shows about eight valley miles of the North Saskatchewan River, in low flow of Sept. 1950 (upper photo) and May 1962, to the same scale. The bed in the main channel is gravel of median size by weight about 3 inches, which means that half the weight in a large sample would contain stones above about 1 pound weight and probably about 10% of the sample weight would contain stones larger than 10 pounds. Shoals are of somewhat finer material. The side-slopes (banks) of the incised channel may be of somewhat finer material than the bed with a binder of sand and sometimes a little cohesive material, or of sandy loam bound by tree and shrub roots, or, where the river hits the valley wall, of clayey or shaley material with an occasional coal seam. The surface of the vegetated flood plain is sandy. Creeks that run only in flood have generally sand beds. Trees on the flood plain are mainly spruce and poplar.

2.33. The river valley is cut about 150 ft. deep in the prairie and is all subject to inundation. The highest recorded flood, which is believed to be about a 300 year one, is about 200,000 cusecs and overtops the flood plain by some 10 feet; approximately, the bankfull discharge is 100,000 cusecs and runs 20 feet deep with the incised surface breadth 700 feet. Slope along the main meandering channel varies from about 0.9 to 0.4 per thousand according to whether there is braiding (para 2.35) or not. The flood plain is overtopped, on an average, about once in 20 years.

2.34. In the short reach under description the meander bends illustrate the sinusoid, the distorted sinusoid, the double-S and a zig-zag pendent loop, abC, on the old creek to the extreme left. Also apparent are chutes, or long cutoffs, such as BC and DE. The valley width is not much more than a major meander breadth. A couple of miles downstream the geology changes and the valley is wider so that there is more room for contortions. Plate 9 shows a recent cutoff that occurred there in a high flood, and the arrows show how a portion of river has reversed its direction in consequence. Obviously another cutoff into the right arm is imminent.

2.35. **Braiding.** A feature that did not appear in the Beaver and the Pembina is braiding, which is the occurrence of multiple chan-
nels. The term braiding is neither precise nor standardised; some writers regard a single channel with a few islands as braided while others, including the author, apply the term only if, at high non-spilling discharge, there is a chain of alternative meandering channels of comparable magnitude.

2.36. The braided situation is shown by the alternative paths from F to K. In 1950 DEfghK was the predominant path, but DEFghK was obviously important; the cross-leakage via Fg seems to have been minor, while GHK seems conspicuous though minor. In 1962 the obviously important path was DEFghK, Fg was still unimportant, but Ffg had been practically cut off. Site inspection during 1963 showed FG eroding vigorously. A map of 1878 showed no loop Efg; instead there was an important channel roughly along Eg and the remaining channels did not seem to deviate much from their 1962 positions. GHK has not changed appreciably since 1878 although its position suggests it ought to have been a cut-off channel for GhK somewhen; site inspection shows that it failed to develop because it runs along an unstable cliff which slides when undercut too much. The loop abC has remained unchanged since 1878.

2.37. Generally, as separate braids meander their flow and sediment and bank conditions change so that they tend to enlarge or shrink relative to their neighbours; the process is crudely cyclic. In the unbraided type of stream multiple channels are unstable so that, if they are created by some accidental natural means, or by engineering interference, one channel will grow till the others shrink to insignificance or silt up altogether.

2.38. In 1950 the erosion situation at hh₁h₂ looked as if the crude double-S, hKLMN, was heading for a contact cutoff (Plate 5) along hN, especially as farmers had cleared trees off the waist. However, by 1962 hh₁h₂ had shrunk and the river obviously preferred the straight path hh₂. The apparent reason is the switch of channels upstream.

2.39. A local feature of engineering interest, because of a proposed plant location on the bank, is that in 1920 the main river channel was along a smoothed line LmN; so it has been eating into the flood plain on the right at about 40 feet a year to form the present bulge. Relative to anywhere else in the photo this erosion rate is large.

2.40. Meander length is rather difficult to assess, but 6,500 feet seems a fairly good fit to the major distorted sinusoids and the
double-S, and might even be held to fit what Efg was in the days when it was a fairly complete loop. Meander ratio and loop chord ratio for the different types of meander are obviously as for comparable ones in the Beaver and Pembina.

2.41. Engineering problems of the river in other comparable reaches are varied and include those mentioned for the Beaver and Pembina. Oil wells located in the flood plain, to be originally remote from direct river attack, came under attack within ten years because of an almost complete reversal of meander pattern. Large gravel removal operations caused changes of bed-level to develop both upstream and down, thereby threatening buried pipelines. The location of water intakes so as not to draw gravel during floods became a problem. Operation of a power reservoir on a major tributary was found to affect the winter ice-jam problem. Plate 10 shows an ice-jam on a similar river of smaller discharge.

2.42. Washita River, Oklahoma, U.S.A. Delta. Plate 11 shows, in 1961, a river that has been readjusting itself at geologically terrific speed in about 7 miles where it runs into an artificial lake that was formed in 1945. New deposits, covered mainly by vigorous growths of salt-cedar (tamarisk), are seen dark throughout the photo; at the mouth the old river bed is 40 feet below the new. The delta front of 1949 for the lake level shown is marked with white figures. The “pool” downstream of “1949” is almost full of sediment but the deposits are hidden by the water; the pool is a tiny part of a very large reservoir. The reservoir has fluctuated between about 607 and 642 elevation during its history because of filling by floods and depletion for engineering purposes.

2.43. A peculiarity of this deltaic river reach is that, if the pool were hidden, there is little to suggest that the incised new river channel is anywhere near a delta, for there is no channel subdivision. This can be explained in terms of the nature and relatively enormous quantity of the sediment load, and the climate. The original river-bed sand was of 0.15 mm, size, and the total sediment load is about 1% by weight of the water against about 0.01% for rivers like the Beaver and Pembina; it contains mainly fine sand and silt. With the Pembina intensity of load the Washita would require 1,500 years to do what it has done in 15 in its delta. The nature of the load is sufficiently close to that of the Beaver’s for it to build up a moderately high flood plain relative to its high flood level, and the subtropical climate induces exceedingly rapid growth of salt-cedar which discourages cutoffs or chutes and aids
fine sediment deposition on the flood plain. In fact, the channel used to continue straight instead of into the overhanging loop near the present mouth, but sediment and salt-cedar have removed all trace of this from the photo and a switch back is now unlikely. Details of this delta up to 1954 have been given by Bondu rant, 1955.

2.44. For comparison with the Pembina and Beaver, very high flood approximates 100,000 cusecs and bankfull discharge of the undisturbed river was about 30,000 cusecs; ratio 3.3. Breadth of incised channel before engineering interference was 200 feet; afterwards it changed to 400 feet, starting at the reservoir and progressing to a dozen miles upstream of the highest lake level by 1961. One explanation of the change of breadth is in terms of the high velocities induced in the new channel when the lake is drawn down and a flood occurs. The original slope was 0.31 per thousand and, in 1961, was still unaffected upstream of several miles upstream of where highest lake level intersects the bed; downstream of this it flattens progressively. The bed-material size now grades down from its original 0.15 mm. through the flattening reach to about 0.08 near the new mouth. The bankfull depth has decreased by several feet in the self-adjusting reach, in sympathy with the increased breadth and the changing slope compelled by the advancing delta.

2.45. The special practical point arising from the relative figures of this exceedingly heavily laden river is that the advance of deltaic disturbances upstream will not develop fast enough to be considered a problem except in rivers with abnormally large loads. Even with them it will develop more slowly as time goes by since the volume of the delta below water-level in an artificial reservoir might vary roughly as the cube of the outward extension of the delta; then the extension into such a lake (which is related to the extension of aggradation upstream of the lake) should be about ten times as far in 10,000 years as it is in 10. Of course, there are lakes where the variation may be closer to the square than to the cube, in which case 1,000 years would replace 10,000.

2.46. Kitimat River, British Columbia, Canada. A more conventional delta than the Washita's is that of the Kitimat River, British Columbia, shown in Plate 12 for about 1950. The climate here is temperate, with about 90 inches annual precipitation, and the area is covered with spruce forest and dense undergrowth. The river is rather like the North Saskatchewan but with larger gravel, a more active bed and far more frequent overspill; fine sand, silt and clay are practically absent. High flood is about 100,000 cusecs.
The delta forms into a fiord; the tide, in the photo, is low. Its relative rate of delta advance per unit discharge would be about one hundredth (in terms of volume) of that of the Washita and vegetation establishes itself slowly on new deposits. So multiple delta channels have been able to establish themselves. Plate 13 is for just upstream of Plate 12 and illustrates D-shaped natural cut-offs that occur tangent to loops; this is where an engineer would put artificial ones to obtain minimum inducement for new bends to start. (See Plate 7).

2.47 A major peculiarity of this river is the effect of log-jams and heavy forest on cutoffs. Through the densest forest in the flood plain there are always a few small creeks—remnants of older river channels. These are unable to enlarge till the loops joining their ends lengthen enough to cause a large head over them. Then the velocity of flood flow through a short creek can rip out large trees and remove them. Ordinary floods are usually ineffective because log-jams block the entrances to creeks and may remain for years before an enormous flood floats them out. If, when this happens, a loop has become large, the cutoff can develop into a major river channel in a day or two. The loop of 1950 that projects to the right (looking downstream) in Plate 12 had been moving outwards for half a century and cut off in 1962 so that now it resembles the cutoff ones of Plate 13. This cutoff increases the tendency for the main delta channel downstream to switch from the left (looking downstream) to the right.

2.48. Engineering problems, since 1950, have concerned how to build and maintain a large industrial operation, with its attendant service and residential areas, mostly within the flood plain. They have been solved economically by foreseeing natural river trends and either avoiding conflict with them or taking action to forestall them while conditions were favourable or relatively harmless. A particular problem has been how to assist the river to continue discharging on the side of the fiord remote from the harbour.

2.49. Kemano River, British Columbia, Canada. Plate 14 illustrates conspicuous braiding. The river is essentially a much more vigorous Kitimat one in a tight mountain valley, with larger gravel and larger load intensity. Active alluvial fan streams enter it from large side valleys, carrying obviously large loads, and the short steep cuts down the mountain sides bring in avalanches in the spring. With large fluctuating load, fluctuating discharges, and log jams there is no chance to settle down to the stability of a
single meandering channel and there are not many places where "the incised channel" has much meaning. The braided channels switch frequently. Along such a river there is little room for a road, railway or transmission line to avoid attack, and engineering problems aim largely at how to devise the most economical direct protective works.

2.50. **General.** The preceding examples should show that rivers, like people, exhibit an outstanding range of individuality but an obvious general similarity suggesting that the major controlling laws are fairly simple. So, if the engineer can find the laws, and will respect them and the individuality, he should not find himself involved in expensive losing battles against nature.
CHAPTER 3

SYSTEM IN RIVER PARAMETERS

3.1. Introduction. A logical step, after acquiring some familiarity with river types, is to become convinced that the various major quantities involved in river behaviour are interlinked by simple dynamical laws and that (a) every engineering or natural "disturbance" (e.g. by a dam or an earthquake) of a river must be followed by a trend back to an equilibrium state and, therefore, (b) knowledge of the laws is essential to river engineering. This conviction cannot be obtained readily from an engineer's own experience since river self-adjustments after a disturbance may require years, decades or centuries to grow and die away; usually an engineer has left the site of his interference with nature before its inevitable sequels have developed enough for the inexpert observer to suspect their causes. Therefore the present chapter will outline salient regularities of river behaviour that have come to the notice of a variety of engineers and geologists since the end of last century; these constitute basic evidence of the operation of physical laws.

3.2. Meanders. Rivers with erodible banks tend to meander; a long straight reach of such a channel is abnormal and transient. The existence of a relation between meander sizes and incised channel breadth had been noticed, and measurements had been made, by the beginning of the century (Jefferson, 1902). The early investigations showed approximate proportionality between breadth and meander belt width. Later ones show approximate proportionality of breadth with belt width and with meander wavelength provided a large range of sizes is studied so that the deviations of individual points become relatively unimportant. Inglis, 1947, 1949, found that size varied as the square root of high flood discharge and that the random deviations of wave-length from the average relation were very much less than those of meander belt breadth. Fig. 3.1 shows a miscellany of data, including model dimensions and observations by Inglis as presented by Leopold & Wolman, 1957(a), but fitted with the line:

\[ M_L = 10b \]  

(3.1)

where \( M_L \) is meander length (i.e. meander wave-length) and \( b \) is
surface breadth of incised channel. Because the range of breadth is very large—about 5,000 times—and the scales are logarithmic, the deviations of points from the fitting line appears small; actually the points fall in a band whose top limit of wave-length is three times that of the bottom. Considering the subjectiveness of assessments of wave-length and channel breadth the scatter is less than one might expect from general experience with river data. Without any formal measurements, the form of eqn (3.1) and a corresponding one with a different coefficient for meander belt width follow from the facts that (a) no means is known for deducing the size of a river purely from its relative dimensions in an air photo and (b) model-makers have never found cause to deviate from their simple practice of making models geometrically similar with prototypes in plan.

3.3. **Breadth and depth.** Common knowledge among those who work on and measure rivers includes (a) large discharge is associated with large dimensions, (b) breadth is greater relative to depth in large rivers than in small, (c) channels that move their beds very actively tend to be relatively broad. However, direct measurements to discover quantitative expressions for these peculiarities are relatively recent and restricted. Leopold & Maddock, 1953, analysed gage site data on one river system at a time to find whether nature imposed, within a system, a fairly simple relation between breadth and depth, on one hand, and capacity of channel (as expressed by an average discharge) on the other; the effects of nature and quantity of bed material, and of nature of sides, had to be ignored. Essentially their final procedure, for every gaging site of a system, was to compute average daily discharge, Q, of a long period, obtain the surface breadth b at that flow within the incised channel, obtain the corresponding mean depth of flow, d, (so that \( bd = \) area of cross-section of flow) and plot b, d separately against Q for every gaging site.

3.4. Fig. 3.2., taken from their work, shows eight fitting lines on double-log paper for seven river systems and one Indian canal system. The selection was made to cover “a diversity of geographic locations and physiographic and geologic types”. Seven systems, including the canal one, indicate the breadth lines sloping at about \( \frac{1}{2} \) and the depth lines at rather more than \( \frac{1}{3} \)—the reported average slopes of all investigations of the Reference are 0.5 and 0.4; one set indicates quite different slopes but still follows an index law. Later comprehensive work by Nixon, 1959, in Great Britain
gives similar results. Other small scale observations have been made. Lacey, 1929-30, found that the index $\frac{1}{2}$ for breadth fitted certain large Indian railway bridges, built in the light of long experience, if high flood discharge was used for $Q$ and the multiplying constant was the same as he had found for self-adjusted canals deriving from such rivers.

3.5. It is important to remember that, although reasonably stable formulas as described represent behaviour of rivers, they cannot be expected to represent physical laws except by accident. Obviously breadth and depth depend on the natures of bed and sides which are ignored in the formulas. So, if these ignored factors are associated, in nature, with the magnitude of discharge their effect must be mixed into the indices of $Q$ in the formulas unless there is some kind of mutual cancellation of effects. However, the existence of fairly stable formulas does indicate that physical causes are present and $Q$ is an important factor.

3.6. Slope. Workers with rivers have known, for long, that large rivers run at relatively flat slopes, and rivers that move large bed material or large quantities of bed material relative to their discharge run relatively steeply. However, attempts to correlate slope with discharge alone over a very large range of discharge (Leopold & Wolman, 1957 (a)) have displayed such wide scatter of points that a fitting curve has little statistical significance or utility. Very recently Blench & Qureshi, 1964, tried plotting a composite variable composed of slope along with breadth and a factor defining properties of the bed material, against an average discharge. They used only data believed to be reliable and to be for moderate or small intensity of bed loads. Fig. 3.3. shows the plot, with model data added, and uses a theoretical band for fitting. Although the discharge range is ten million times there is hardly any deviation of points outside the band. Points for large bed load intensity, or for small gravel rivers where the size of bed-material relative to depth exceeds some limit, will fall above the band. High position is also associated with marked tortuosity. Ignoring theoretical explanations and special circumstances for the present, the fact remains that there is obviously a simple relation between slope and other parameters of the flow in a self-formed river.

3.7. Constitution of bed-sand. The most exact quantitative regularity of a sand-bed river is in the constitution of the sand that is active on the bed. Petrologists (Krumbienen, 1938) have found that many sediments in nature, whether loose or turned into rock, have
a remarkably systematic constitution as long as their sizes are in the sand range. Constitution is tested by sieving the various sizes of a sample of sediment and plotting grain sizes against the percentages by weight of the sample coarser than each size in turn; details will be found in references on soil mechanics. If the plot of mechanical analysis is performed as in Fig. 3.4 on a special graph paper, called probability paper, that statisticians have devised to test for what they call the “error function distribution,” a good straight line is obtained provided the grain size is measured by its logarithm or, what is essentially the same, by its Tyler sieve number —Tyler sieve numbers increase by one each time the opening is halved, so that they actually constitute a logarithmic scale of sizes to the base two. The straight line indicates that the error function distribution is followed. Mathematicians have demonstrated that certain simple postulates about causes of variations will lead to the error function distribution. It is found in many quantities that scatter about a mean, e.g. precision measurements as in surveying, precision manufacture as in screws, examination marks of large classes, annual peak floods of some rivers, and even—an example given by statisticians—chest measurements in regiments. Apparently, then, there are quite definite causes for the sediment constitution.

3.8. In Fig. 3.4, from Blench 1953a, sands from the active bed of a large estuary are analysed non-dimensionally by expressing each particle size in a sample as a fraction of the median (middle size) of the sample; thus, the particle size plotted against 50% is always 1.0. The ordinate is the logarithm of particle size. The samples were collected by non-technical personnel from some 30 feet of water by dragging a metal pot along the bed and were taken at 2 mile intervals over some 30 miles at various times; the sieving intervals were closer than usual. General experience shows that sands that have been moved in dunes follow the logarithmic error function distribution closely; sands in relatively dead water may contain fines that upset the distribution. Sands of desert dunes and of wind-tunnel experiments have been analysed by Bagnold, 1960, to a different system; reanalysis on a chart like Fig. 3.4 shows them to be logarithmic “normal” (which is the popular synonym for “error function”) also but to have a significantly wider range of sizes within samples than is found in water-worn sand. As nature certainly does not provide sand particle sizes from catchment erosion in the proportions found in bed samples, the logarithmic normal distribution probably represents just the right
proportioning of different sizes of certain shapes to permit them to move in a group. When natural active river bed sand is used in a model it does not change its constitution; unnatural material—such as from a pile of mixed dredgings—does.

3.9. Constitution of bed-gravel. Gravel samples taken at random from river beds do not usually follow the simple distribution found in sands. However Qureshi, 1962, and Kellerhals, 1963, have found a good approximation to the distribution in deep channel and, particularly, in rivers that take out from lakes so that the finer materials have been washed away long ago; the range of sizes is larger than in sand. The simpler behaviour of sand may be attributed to the fact that it is the final product of natural elutriation so possesses a relatively small range of sizes on which the water can act during transport.

3.10. Bed-formation. Usually, in sand bed rivers, the active bed moves in dunes as in Plates 21, 22, resembling the wind-blown dunes of the desert; at speeds above the critical (i.e. above the velocity of wave propagation in the water) dunes may be absent or may assume a flat sine-wave form that moves against the flow although, of course, the movement of sediment continues downstream as usual. Until sonic sounders came into use, there was considerable technical belief that dunes did not occur in rivers, although their existence in canals of several thousand cusecs capacity must have been known from the dawn of irrigation in India. Records from sonic sounders have been reported by Pretious & Blench, 1950, Carey & Keller, 1957, and Neill, 1965 (a). Quantitative work on river bed forms is insufficient to merit a statement here but laboratory work is growing and is showing details of dune forms and the circumstances surrounding them, paras 10.4-10.7. Dunes are unlikely to form, or to be conspicuous, in gravel rivers, except under special conditions, and technical difficulties have limited observation. For the present chapter the important point is that, in sand bed rivers, the normal bed-form is the obviously systematic one of desert sand-dunes.

3.11. An application. With nothing more than preceding facts about observed river behaviour a fair forecast can be made of possibly undesirable consequences of proposed interferences with rivers.

**Problem 3.1.** A sand-bed river, A, of the type shown in Plate 4 approaches another of similar type, B, and the land between the two is similar to that of both their valleys. Diversion of A
into B, for irrigation use, would be cheap and, from a water rights viewpoint, unobjectionable. The direct slope in the connecting channel would be about twice the slope straight down the valley of A. The irrigation officials proposed a straight diversion cut which, for economy, would have half the breadth of the incised channel of A; its length would be about ten meander lengths of A. They reckoned that the diversion would probably stay straight, that they could patch sides here and there if it did not, and that the fines in the sand would probably wash out so that the friction in the channel would become more and this would make up for increased river slope. What possible developments should they investigate before proceeding?

**Answer.** Fig. 3.2 shows a range of breadth, for a given discharge, of little more than two times among the eight systems analysed; so halving breadth is an extreme measure and should be expected to result in marked bank erosion. Fig. 3.3 shows a bandwidth of four-times to cover straight channels of very little load intensity at the bottom and tortuous ones of moderate intensity at the top. The length along the loops of river A is about 2.5 times the straight length down the valley, so the slope of the proposed diversion is $2 \times 2.5 = 5.0$ times that along the loops of A. Therefore, if the data of the diversion of A were plotted on Fig. 3.3, the point would be displaced upward by more than a bandwidth from the point for A. That is, the diversion would start to run as a relatively heavily-laden river. Therefore it would pick up load from its bed and start to flatten its slope. According to paras 3.7, 8 there is no evidence that a sand-bed river would develop a different type of grain distribution than the logarithmic normal; Bagnold's experiments with a wind tunnel indicate that it would not. From a slightly different viewpoint one might argue that the diversion would have to develop, eventually, into exactly the same form as river A, since all its permanent circumstances are the same. So eventually it would meander (para 3.2) like A and its eventual data would plot on Fig. 3.3 very close to the position of the present A point. But, if the diversion were constrained, by rip-rap on the sides, to remain straight then the eventual plotted point would be lower than for A (para 3.6), which means that the final slope would be less than that of A. A rough estimate can be made of eventual possibilities. Suppose the river slopes $1/3,000$ along the flow, or $2.5/3,000$ along the straight, and that
a meander length is 3,000 feet. The drop initially in the diversion is $2 \times 10 \times 2.5 = 50$ feet. If the diversion were allowed to meander then, eventually, the slope along the straight would return to $2.5/3,000$, with a 25 foot drop instead of the 50 given originally, and the whole river upstream to the first control point would degrade by $50 - 25 = 25$ feet so as to regain its natural slope. If the diversion were maintained straight by protecting the sides, but not the bed, then the slope would become less than $2.5/3,000$—possibly $1.25/3,000$—so the drop over the diversion would become 12.5 feet instead of 50, and the degradation at its head and in the whole river upstream would be $50 - 12.5 = 37.5$ feet.

3.12. The preceding problem assumes that the diversion and the river upstream remain subject to the definition of para 1.1; in a real case they might cut down to inerodible material and either develop drops or merely cease to erode further. The intermediate developments before a final state was reached have not been considered though one of them would be a rise of river downstream, for several years, as a consequence of the products of erosion from upstream. However, the point illustrated is that quantitative knowledge of the general facts of behaviour of rivers, when they have acquired a relatively steady state, can give an assessment of the magnitude of likely long-term self-adjustments after proposed engineering interference; the type of study required before making a decision then becomes apparent. If the ultimate adjustments appear large then the early ones are likely to be significant. Shulits, 1955 has described major self-adjustments of the Rhine, Germany, after straightening. Paras 1.5 to 1.16 of the present text also illustrate.
CHAPTER 4

INITIAL TERMINOLOGY AND CONCEPTS

4.1. Introduction. In the preceding chapters several terms and concepts, related to rivers’ self-adjustment to suit laws of nature, have been introduced informally and naturally, rather as they would in the long acquisition of field experience. As future chapters will be more technical some of these terms and concepts will be specialized and discussed now, and some new ones will be added. The reader may prefer to read cursorily at first, and to study later as need arises.

4.2. Regime Channel. Unless stated otherwise, every natural or artificial channel in this text is assumed (para 1.1) to have a bed of noncohesive material that moves above some stage of flow; the sides need not be so restricted. Such a channel has come to be called a “regime channel”.

4.3. Regime. The noun “regime” applied to a channel or channel reach is analogous to “climate” since it implies a behaviour that is appreciated in terms of many fluctuating factors whose average values, over a sufficient period, are either steady or change relatively slowly. Such a slow change is called “secular” (Latin, saeculum, age, span of time). The mind finds no difficulty in visualising a climate or a regime as a relatively steady state of large erratic fluctuations, though statistical technicalities are involved in defining, exactly, “a sufficient period” and “a secularly changing mean”. THE WORLD BOOK DICTIONARY (1963) defines “climate” (Greek, klinein, to incline) as “the kind of weather over a period of years, based on conditions of heat and cold, moisture and dryness, clearness and cloudiness, wind and calm. . . .”. So “regime” may be defined as “the behaviour of a channel, over a period, based on conditions of water and sediment discharge, breadth, depth, slope, meander form and progress, bar movement, etc. . . .”. Unconventionally, but descriptively, it could be called “the climate of a channel”.

4.4. In-regime. The term “in regime” will be used always in the technical sense that the regime, over a period, does not change. Applied to a river there is an implication, as in saying that the climate does not change over a period, that the period must be
several years to permit a proper judgement; for one major cycle of water and sediment discharge events occupies at least a year. For special technical purposes (Chapter 9) the period may be less, for reasons that would have to be explained in the context.

4.5. **Specific gage.** There is no single sufficient test whether a channel is in-regime. However, for rivers, the most powerful single necessary test is to plot curves of “specific gage” against time; if the curves neither rise nor fall consistently the channel is in-regime in the vicinity of the gaging site for most practical purposes.

4.6. A “specific gage” is the gage for a specific discharge—say 100,000 cusecs—at a gaging site and is obtained, normally, from the stage-discharge curve of a season or year. A specific gage curve normally plots specific gage against years. Fig. 4.1 shows a set of specific gage curves for a site just downstream of the Lloyd Barrage on the Indus River; the specific gages from stage-discharge curves for both rising and falling stages are used; but the time intervals are one year in each case. Obviously the river was not in regime. The object of the curves was to show the relatively sudden regime changes due to barrage construction and later extension, each followed by a relatively slow trend (or secular change of regime) towards a new steady regime at a higher elevation than originally, due to sediment exclusion from the canals and reduced river flow.

4.7. A channel might change its breadth considerably without specific gage curves indicating that anything had happened. A climatic parallel is that widespread irrigation might alter the humidity of a tract of country obviously, yet annual precipitation graphs might show no trend. This insufficiency of any single test has been mentioned already.

4.8. The reason for plotting several curves for one site, to cover the whole range of discharge as far as practical, is that individual curves may show different trends. In one practical analysis low stage curves rose steadily for a decade, the intermediate ones dropped steadily, and high flood stages remained unaffected. This behaviour was at a gaging site well upstream of a reservoir and was explicable in terms of the general river rise upstream from a growing delta, the increase in breadth due to draw-downs caused by the reservoir fluctuating out of step with the river, a couple of cut-offs occurring in different years at different distances from the gaging site, and the relatively large flood spill.

4.9. **The equilibrium concept.** According to regime theory (para 1.3 and Chapter 6) the factors in a regime are determined by
dynamical laws. Therefore an in-regime system is one in dynamical equilibrium and, as shown by Chapter 5, the equilibrium is normally stable—that is, it restores itself after a disturbance if the causes remain unaffected by the disturbance.

4.10. If the dynamical causes (e.g. discharge) of regime factors fluctuate about steady or secularly changing time-mean values (over a period) then the regime factors (e.g. breadth, meander length) must do likewise, and conversely. More generally, if any time-mean regime factor suffers a change then the other time-mean regime factors in a physical law in which it appears must also suffer change, in general. For example, if engineers remove sediment load by trapping it in a reservoir upstream, or force a meandering channel between straight revetted banks, then all interconnected time-mean regime values are liable to suffer change till a new in-regime condition is attained; in fact, in both cases, the slope must tend to an ultimate smaller value than existed immediately after the interference.

4.11. Obvious though the equilibrium concept may seem, dynamically, it is denied tacitly by agencies that alter river regime factors without allowing for the inevitable self-adjustments.

4.12. **Mean and equivalent uniform values.** The term “mean” has been used in the preceding text with the obvious, and statistically correct, general implication of “some central value of a fluctuating quantity”. When a rule is devised for computing it from the fluctuating quantities then the mean is named to indicate the rule, e.g. arithmetic, harmonic, root-mean-square, median. However, if the physical law of the fluctuating quantity is known then a significant “equivalent uniform value” can be calculated, instead of a simple statistical mean, to suit the problem in hand. As a simple illustration of an equivalent uniform value’s definite intention and dependence on the problem, consider square plates of sides 1, 2 and 5 units long. The equivalent uniform side, from the viewpoint of the length occupied by squares laid side by side, is \((1+2+5)/3 = 2.67\). From the viewpoint of painting them the equivalent uniform square area is \((1+4+25)/3 = 10\) square units, and the side to correspond is \(\sqrt{10} = 3.16\) units.

4.13. A statistical mean has no physical significance in its own right although, in a particular context, it may happen to have it. There is occasional non-technical misconception that a mean, particularly an arithmetic one, must have some special dynamical significance.
4.14. **Equilibrium values of variables.** Implied in the equilibrium concept is a belief in the existence of steady or secularly changing equilibrium values about which the instantaneous values of regime factors fluctuate; these equilibrium values are linked by laws. Strictly the equilibrium values are equivalent uniform ones and cannot be computed without knowledge of the laws affecting the variations about them. In well controlled laboratory experiments fluctuations are so small that any ordinary mean is practically the same as any other or as the equivalent uniform value. In the canals on which regime theory equations used in this text are based, fluctuations within the range of values relevant to self-adjustment were small enough to permit equilibrium values to be assigned to designed full supply conditions with an error believed to be small. River fluctuations in the range of self-adjustment may be enormous so, although equilibrium values exist, there is no obvious way of determining them exactly. However, as described later, a variety of practical problems can be solved with useful accuracy by posing them suitably in terms of mean values of regime factors that are regarded as multiples—different for different cases—of equilibrium values.

4.15. **Dominant, formative, or regime discharge, sediment discharge, etc.** The idea of an equivalent uniform causative quantity, such as discharge or sediment charge, has come to be expressed popularly by the terms “dominant” or “formative”. The terms appear to have been originated by C.C. Inglis, 1949. He applied the term “dominant discharge” to the steady discharge that would produce the same meander length as a natural sequence of discharges; the steady discharge result was from models and the natural one from rivers of one rather general type, and comparison of the formulas for both permitted the relation of dominant discharge to the discharge statistic used for the rivers. It is to be noted that there is no obvious reason to expect an equivalent uniform discharge calculated from one phenomenon—for example meander formation—to be exactly the same as from another such as self-adjustment of slope.

4.16. The term “regime” may be used instead of the preceding two, since regime factors are measured, ideally, by equivalent uniform values. In general literature the terms may be attached loosely to arbitrary means, but with the implied hope that these means are fairly constant multiples of true equilibrium values.

4.17. **Degrees of freedom or self-adjustment.** The number of in-
dependent dynamical equations (independent in the sense that none can be deduced from the others) necessary for the quantitative solution of a regime problem must equal the number of relevant dynamical laws. Facility in deciding the number of equations for a particular problem may be aided by thinking of standard cases in terms of degrees of self-adjustment or, borrowing a term from kinematics, degrees of freedom; to each degree corresponds a law.

4.18. For example, from a kinematic viewpoint, a steady discharge can run uniformly at any depth in a given infinitely long uniform laboratory flume with rigid boundary. However, nature restricts this freedom by imposing a law of hydraulic resistance (expressed by the flow formula of rigid boundary hydraulics) which specifies that only one uniform depth can occur permanently. If a device, such as a gate, is dropped suddenly into the flow, so as to compel a different depth locally, the flow will adjust itself rapidly to achieve the original depth again except near the disturbing gate. The system is said to have one degree of freedom, since one imposed condition removes all freedom, or to have one degree of self-adjustment (with respect to depth). If the flume is now run with a fixed water discharge and a fixed sediment discharge, with matters arranged so that the sediment can settle out to form a moving bed, then, kinematically, any of an infinite number of depths and an infinite number of bed-slopes (greater than the flume slope) can be imagined to occur. But nature imposes the law of hydraulic resistance and a law of bed-load transport so that only one depth and one slope will persist. Insertion of the gate, as before, will result in a relatively slow (since sediment movement and deposition are slow) reversion to nature’s depth and slope except close to the influence of the gate. This system has two degrees of freedom or self-adjustment since two imposed conditions acting on two factors were required to remove freedom; its self-adjustment is with respect to depth and slope. If the flume sides were made erodible and the experiment were started with the breadth small enough for side erosion to occur, but meandering were prevented, then there would be three degrees of freedom or self-adjustment; a third law concerning the balance between erosive resistance and erosive attack would exist. If meandering occurred a fourth equation would be needed and the system would have four degrees of freedom. Replacement of steady quantities by equivalent uniform values of fluctuating ones does not alter the argument.

4.19. Apparently most rivers have four degrees of freedom, so
cannot have their regime problems solved quantitatively without four independent dynamical equations; others may be needed to specify imposed conditions. The most general artificial canal problem has three degrees of freedom and needs three independent equations.

4.20. **Water-sediment complex.** Because dynamical explanations of regime must rest on the interaction of water and sediment, moving together, references to "the water" alone can mislead. The term "water-sediment complex" has been adopted, therefore, for the combined water-sediment flow; "complex" or "water-sediment" might be used.

4.21. **Bed-load, suspended (wash) and total loads.** These terms have long conveyed adequate meaning in general qualitative use although, for quantitative work, there is no accepted definition or method of measurement that will separate the quantities under all conditions. Unless specially stated only the general qualitative usage will be employed in this text. That is, "bed-load", at any stage of flow, means the quantity of bed material (judged visually or by some simple sampling device) that passes a channel section per unit time by rolling and by being whisked into suspension and deposited repeatedly and sporadically. "Suspended (wash)" load is the quantity of sediment that passes a channel section per unit time without ever resting on the bed, so is virtually washed through the channel. "Total load" is the sum of bed and suspended loads. All the quantities fluctuate with time, even in channels that appear to run steadily, so the values used are long-term means. Bed-load may be of different materials at different stages of flow.

4.22. Channels with large total loads often move material that grades down from bed-material size to clay size without obvious discontinuity; then there is no way to distinguish the division between bed and suspended load. For ordinary purposes this is no more serious than inability to decide where the green merges into the yellow in a rainbow when the important matter is to recognize that both colours are present in bands.

4.23. **Clay, silt, sand, gravel.** Different authorities define these materials differently, in terms of size ranges. For the usually rough identification purposes of this text clay is material finer than about 0.002 mm., exhibits cohesion in soils, shows Brownian motion when in suspension in water, and takes more than half an hour to settle in a glass of water. Coarser materials lack cohesion. Silt is non-cohesive material from about 0.002 mm. to 0.02 mm., is not gritty when bitten, exhibits marked capillarity, and takes at least a couple
of minutes to settle in a glass of water. Sand is coarser than silt
with a top limit of size beyond which the terminal settlement vel­
city of particles in the water of the problem varies as the square
root of particle size; so the top limit is about 1 mm., depending
slightly on temperature. Gravel is coarser than sand. Literature
sometimes refers to “cobbles” and “boulders” with the rather vague
meaning of “large” and “very large” gravel; a couple of very dis­
cordant definitions by size have been advanced.

4.24. Hydraulically there would be advantage in classifying ma­
terials, at least partially, in terms of terminal settlement velocity,
Vs, in water. Spherical particles of specific gravity 2.65, in water,
at room temperature, have \( V_s \) proportional to the squares of di­
diameters less than about 0.10 mm. and to the square roots of diameters
more than about 1.0 mm.; between these limits no simple index
law fits.

4.25. **Charge and concentration.** Practical convenience is suited
by thinking in terms of the discharge of liquid in the water-sediment
complex, and the proportionate discharge of sediment; for example,
10,000 cusecs with 3 parts per hundred thousand of bed-load dis­
charge. As the sediment cannot be measured conveniently as a
volume, and a ratio is most useful when non-dimensional, the pro­
portion is expressed as a ratio of weight discharges. Thus, the
“charge” of any portion (e.g. the sand, or the bed-load) of the
sediment load is defined as the weight (in air) of that portion
of the sediment flow per second, divided by the weight of the
water flow per second.

4.26. Concentration at a point means, as in other branches of ap­
plied science, the quantity of sediment per unit quantity of water
in an infinitesimal quantity of water at that point at a given time;
it has no necessary connection with charge. Concentration in a
finite quantity of water would be the quantity of sediment there
at a given time, divided by the quantity of the water, so would be
a space-average. As concentrations in channels usually fluctuate
violently with time the value normally quoted is a time-average
of the space-averages over a period. In river work there is usually
no significant difference between concentration of sediment per
unit quantity of water and per unit quantity of water-sediment
complex; the quantities are taken as weights in this text. The
physical difference between charge and concentration is emphasised
by imagining the sediment was replaced by a shoal of fish swim­
mimg upstream. The charge of fish would be negative; the con-
centration could be measured by trapping a sample of fish between two gates, dropped suddenly, and dividing the amount of fish by the amount of water. Charge and concentration can be the same if every particle of sediment moves with the same velocity as the water in its vicinity.

4.27. **Phase.** A major potential source of formula misapplication in applied fluid mechanics is neglect to ascertain the "phase" (Greek, phanein, to show or appear), or state of a flow. Phases are usually shown either by differences in visual appearance or by differences in the curves fitting plotted data relevant to the flow; either test may be inadequate by itself. The universally known example of phase differences in elementary hydraulics is demonstrated by the conventional friction factor diagram for Newtonian liquid flow in circular pipes. Fig. 4.2 shows such a diagram with lines representing some standard pipe data and some canal data. It shows that, in turbulent pipe flow, there is one phase fitted by a simple formula containing only Reynolds' Number, a second phase fitted by a simple formula containing only relative roughness, and a third phase which would merit subdivision if enough were known about it. This last phase's formula contains Reynolds' Number, relative roughness and something to do with the unmeasurable roughness shapes and distributions. The three phases are not obvious from visual inspection of the flow.

4.28. With a water-sediment complex one has to cope with the phases of pure fluid flow in channels—which, because of the free surface, exceed those in pipes—combined with three different phases in settlement velocity of sediment. On top of this is the difficulty that a sediment load is usually of many sizes, so the phases will be mixed. It is not surprising, therefore, that laboratory work, with no suspended load, shows four obvious phases of bedload transport to which experts add a couple. Regime formulas, based on sand canal observations and amended practically to include some laboratory results, are presented in this text with repeated warnings about the limits of applicability, and their application to rivers contains further warnings. These admonitions serve somewhat the same purpose as the friction factor diagram whose routine use draws perpetual attention to phases. Chapter 10 treats phases in detail.
CHAPTER 5

IMPORTANCE OF INDO-GANGETIC IRRIGATION CANALS

5.1. Introduction. Quantitative analysis and explanation of river behaviour are the concern of this text and require formulas expressing the controlling dynamical laws. Fluid dynamics cannot deduce these laws rigorously at present so they must be obtained, as closely as possible, from observation. Unfortunately, observation on rivers themselves is handicapped by factors including the difficulty or impossibility of measuring all the relevant variables, the large number of quantities that vary at one time, and the impossibility of controlling quantities. On the other hand laboratory experiments that do control factors, so that only one or two vary at one time, are also severely handicapped. If they are flume experiments they suffer, inter alia, the potential liability of all very small scale work to operate in a phase (para 4.27) different from the prototype's; they do not cover a large enough range of certain variables to permit confident extrapolation to river size; they have only two degrees of freedom against the four for rivers (para 4.18); they do not reproduce natural bed-material (para 3.7) readily. If they are models of rivers their small scale and artificiality make them suspect for quantitative work, although their qualitative value is high; they are time-consuming and expensive.

5.2. The irrigation canals of the Indo-Gangetic Plain are virtually controlled rivers of quite a general type. They have three degrees of freedom and a tendency to acquire a fourth. They operate in one phase which appears to be the ordinary one of all sand rivers and many gravel ones. They occur in groups within each of which certain factors are practically constant. Where flume variables have small range, they have large range (Table 5.1).

They were built out-of-regime and forced engineers to permit them to come into-regime, so they demonstrate, relatively rapidly and actively, the self-adjustment that, in nature, is relatively slow and is usually inferred from geological evidence. Their sediment is natural river material (para 3.7) that has adjusted its constitution to natural processes of disintegration, attrition and elutriation till, on the bed, it has acquired a remarkably simple particle-size distribution. Their routine measurements are of fair accuracy and
Table 5.1. Comparative ranges of some regime data

<table>
<thead>
<tr>
<th></th>
<th>Indian canals</th>
<th>Flumes of Gilbert</th>
</tr>
</thead>
<tbody>
<tr>
<td>D mm</td>
<td>0.10 - 0.60</td>
<td>0.30 - 7.0</td>
</tr>
<tr>
<td>Grading</td>
<td>Log. prob.</td>
<td>Uniformised</td>
</tr>
<tr>
<td>C per $10^5$</td>
<td>0 - about 3</td>
<td>0 - 3,500</td>
</tr>
<tr>
<td>Suspended</td>
<td>0 - 1%</td>
<td>Nil</td>
</tr>
<tr>
<td>Water temperature</td>
<td>50 - 86°F</td>
<td>Prob. 55 deg. F</td>
</tr>
<tr>
<td>Sides</td>
<td>Clay, smooth</td>
<td>Wood or glass, smooth</td>
</tr>
<tr>
<td>b/d</td>
<td>4 - 30</td>
<td>1 - 50</td>
</tr>
<tr>
<td>$V^2/d$ ft/sec²</td>
<td>0.5 - 1.5</td>
<td>7.5 - 250</td>
</tr>
<tr>
<td>$Vb/\nu$</td>
<td>$10^2 - 10^8$</td>
<td>$10^5 - 10^9$</td>
</tr>
<tr>
<td>Q cusecs</td>
<td>1 - 10,000</td>
<td>0.1 - 1.0</td>
</tr>
<tr>
<td>Bed phase</td>
<td>Dunes</td>
<td>Dunes, sheet, antidunes</td>
</tr>
<tr>
<td>d/D</td>
<td>&gt;1,000</td>
<td>6 - 6,000</td>
</tr>
</tbody>
</table>

have been made, analysed and discussed for decades by Government agencies including research departments.

5.3. Accordingly these canals will be treated as the main "laboratory" of the inductive science of regime theory. The immediately following paragraphs will outline their "experimental set-up" and principal qualitative findings. In later chapters the quantitative findings, supplemented from indoor laboratories, will be discussed, with applications to canal problems that have special relevance to river ones. Finally, rivers will be considered as canals from which restrictions have been removed progressively, regime theory formulas and findings will be used as far as justified by cautious tests, and the growing, but still limited, information on formulations for phases outside the regime-theory one will receive attention.

5.4. Physiography of the Punjab. The Punjab, before partition in 1947, was India’s major irrigating province with irrigation greater than that of the U.S.A. The growth of regime theory is very closely associated with it, its conditions are typical of Indian irrigation one generally, and the author knows it well. Therefore, as a preliminary to the history of development of regime ideas and theory, its relevant physical setting will be described. The province is an arid plain, out of which mountains rise abruptly in the north. Its 100,000 square miles are traversed by the Indus with its tributaries the Jhelum, Chenab, Ravi, and Sutlej; “punj” means “five”, and “ab” means “water”. Winter discharges are very small; but as snow melts in the mountains in spring the rivers rise steadily and,
when monsoon rains fall in the late summer and autumn, great flood peaks are attained. The Jhelum and Chenab have each attained records of nearly 1,000,000 cusecs; this discharge is comparable with high discharge of the Mississippi at Vicksburg, U.S.A.

5.5. The summer waters are heavily charged with sediment of grades varying from clay size up to sand of about 0.6 mm. in the plains reaches; the various grades are present simultaneously, but in different proportions at different times, and with finer maxima as distance from the mountains increases; gravel occurs near the foothills. The sediment quantities vary greatly; something under 1 per cent by weight is a fair average for the whole year, and the figure is practically zero during low supply. The Sutlej is considered, for calculations of sediment deposition likely to occur in reservoirs, as having sediment-load intensity comparable with that originally in the Colorado River near Hoover Dam, U.S.A. Its load is estimated at 35 million tons per year, capable of covering about 20,000 acres (i.e. 31 square miles) to a depth of one foot; as the water could cover some 3,000,000 acres to a depth of one foot, there are about 11 tons of sediment per acre-foot of water, making the percentage by weight about 0.8. Judging from slopes, and from the exceedingly small bed-load charge entering canals, the bed-load charges must be of the order of a few hundred-thousandths by weight.

5.6. The material of the plains is what one would expect from the present-day sediment of the rivers. The surface soil would be classed, agriculturally, as a silty clay loam, occurring in a definite crust, under which are found great depths of clean river sand—grades from about 0.1 to 0.6 mm., with a mean about 0.25 mm.—containing lenticles of clay, and layers of gravel and nodular limestone. Good bricks can be made from the soil of almost every village.

5.7. In most of the province the rivers meander in a “khadir” area between the so-called high banks up to about 200 ft. high and a few miles apart. In the south the high banks die away and the situation approaches that of the neighbouring province of Sind where artificial banks have had to be built because the high floods spill over the plains.

5.8. The climate is similar to that of California’s Central Valley. Winters are cold enough to produce frost at night; summer temperatures rise to as much as 120°F in the shade. Trifling rains fall around Christmas; the main monsoon rainfall occurs between July
and September and is violent while it lasts. Rainfall near the foothills is of the order of 25 in. a year, but tapers off rapidly as the plains are entered, and is about 5 in. per year in the south. These conditions impose river water temperatures varying from about 50° to 85°F during the year. Evaporation from pans on an extremely hot and windy day may be ¾ in.; annual evaporation in the very arid districts is of the order of 10 ft. per year.

5.9. The canal system. A canal system may be visualized as the stem, veins and capillaries of a leaf. The river (twig) is provided with a low barrage which, being incapable of storing appreciable water, passes a share of sediment load into the system. The main line (stem) divides terminally and laterally into branches (veins) of a couple of thousand cusecs (cubic feet per second) each. These divide again into distributaries or laterals (capillaries) a mile or two apart, and may further sub-divide into minors or sub-laterals. Only the distributaries and their minors are provided with outlets or turnouts, at intervals of a mile or two, to supply water to the farms. Within the farms a network of watercourses or field ditches (sub-capillaries) takes water to the fields. A fairly large system would distribute some 10,000 cusecs and may cover a leaf-shaped area of 150 by 50 miles.

5.10. Growth of Punjab canal systems. Inundation canals have existed along the rivers from time immemorial; their interest is mainly historic. The first important new systems, based on river barrages and run perennially, are the Upper Bari Doab and the Sirhind from the foothill reaches of the Ravi and Sutlej rivers respectively. They were built in the 1880’s, of comparable size, but the former received more publicity for its sediment troubles. Each had a capacity of about 9,000 cusecs. New systems followed at about 20-year intervals till, by 1947, the last desert had been put under irrigation by run-of-the-river water and mountain storage schemes were being planned. The interlinking of rivers was started by the Upper Chenab Canal of about 13,000 cusecs and the Upper Jhelum Canal of about 9,000 cusecs in 1914; by 1947 all the Indus tributaries were inter-connected, some multiply. Since then, in Pakistan, link canals are being constructed and planned with international aid to overcome the difficulties caused by partitioning the Province between India and Pakistan across the water-sheds. Blench, 1951 (b), 1957, has given some details of the pre-1947 period and the growth of regime theory.

5.11. Data. From the earliest times every canal of a system pos
sessed a gage-discharge curve that was revised at short intervals. Before current meters were introduced discharge measurements were by velocity rods (each weighted to cover most of the depth of the vertical on which it recorded); the method was slow but more consistent than by current meter and possibly less subject to turbulence error (Blench, 1936). Meter flumes were introduced gradually; the theoretical ratings of stream-lined long-crested ones were probably as good as current-meter tables. Every channel's daily head gage was recorded, with the related discharge, daily in divisional and subdivisional headquarters. Routine discharge observations were started, under a special organization, in the 1920's for all river barrage sites and main line canals; for inter-Provincial reasons main line discharges were observed by current meter daily even when long-crested weirs made the work technically unnecessary. Also in the 1920's routine suspended load observations started at barrages, and Irrigation Research Institutes were created. Nation-wide research coordination was through the Central Board of Irrigation, Government of India, and the essence of most useful information on canals and rivers was condensed into its various Technical Publications and Annual Reports (Tech.). It is now superseded by the Central Board of Irrigation and Power (New Delhi) for India and the Water and Power Development Authority (Karachi) for Pakistan.

5.12. The canal section. The typical Indian canal has a sand bed, covered with dunes as in a laboratory channel (Plate 21), and berms (Fig. 5.1) of silty clay loam. If the channel is “in cut”, the berms are cut in the natural soil; if it is “in fill” then the berms are deposited out of the suspended sediment load, which is considerable during the river flood season and makes the flow chocolate-coloured and slimy to the touch. The natural deposition of berms and bed from the sediment load may be regarded as a model demonstration of how a river builds a flood plain (para 2.9). In (a) of Fig. 5.1 is the final product; the original bank has been deliberately built to accommodate a larger channel section than will form, and the channel bed has made itself from the sand of the sediment load while the berm has made itself from the fine sand, silt, and clay, so is virtually hydraulic fill such as used to be popular in earth dam construction. In practice the berm formation requires some assistance to ensure that a meandering canal does not develop, and to accelerate depositing. So brushwood spurs are made, as in (b), at intervals of a couple of times their projection—five times is the limit of effectiveness—with
the result that a straight but serrated berm forms as in (d). In cross-section the berm has a lip adjacent to the flow, as in (b), when it first forms; this peculiarity exists also in flood plains built from loamy deposit. As a ragged canal deteriorates rapidly, and as berm hollows grow aquatic weeds, condition (b) is remedied by cutting earth from the side slope and depositing it in the hollow as in (c); the space left by the cutting fills in a few days in the dirty water season. A couple of such cuttings and fillings give dry grassy berms; then the serrated edge is trimmed to leave a perfectly regular canal that is practically breach-proof and will not deteriorate except through long neglect.

5.13. The power of suspended load containing silt or clay to deposit and form banks is further demonstrated by the standard method of repairing a breach when such a load is available. The method is to build a ring bank round the breach, as in (e), so that the enclosed space will fill itself with fine sediment; then the bank is rebuilt over this natural hydraulic fill. Likewise, if a canal has deteriorated by grassy berms growing out, dropping clods into the canal and starting local erosion round the clods so as to cause side-slope collapse, then a standard remedy is to cut the berms back severely to give the canal an increased breadth; suspended load will deposit on the cut banks to make them grow in again regularly and re-establish the original breadth.

5.14. A digression is merited here to enumerate the practical advantages of designing earth canals with berms. A naturally deposited berm is highly breach resistant, and will hold for some time after the bank has developed a small fissure; it is free from alkali; it has a small but appreciable effect in reducing seepage losses; it is a line of defence against the depredations of burrowing animals. Even an artificial berm, in a channel in fill or in cut, has the advantages that it allows for imposition of breadth changes within regime limits without interfering with the bank (whose top is often a road), allows a margin for changing breadth without interfering with the bank when designed discharge is increased, allows bank alignment to be smooth when depth of cut varies, and prevents material washed down by rain from entering the channel and causing irregularities that produce vortices that eat into the bank and cause accelerated deterioration. In lined channels a berm has few advantages and may be a nuisance if not sloped upward from the lining edge.

5.15. The standard section may be taken to be trapezoidal, with side-slope about two upon one. At first this may seem inconsistent
with the dunes of sand on the bed—they are often very conspicuous and may look, during closure, like the waves of an angry sea. Yet Fig. 5.2, is a typical plot of a small canal section from survey data. The explanation is that dunes run generally at right angles to the flow, so a surveyor does not traverse them from crest to trough when observing a cross-section. Strictly he should survey an area to obtain mean bed; actually the averages of many sections at different places and times lead to the same result. The standard section, then, represents a mean bed level of sand, very nearly horizontal in straight canals, and replaces the slightly curved cohesive sides by straight lines.

5.16. Some authorities prefer to regard the standard section as elliptic since, especially in large channels (which are relatively shallow), a slight dishing of the bed is distinct and an ellipse is a good formal geometric average fit. However, where sides are cohesive the discontinuity between steep cohesive banks and flat duned noncohesive bed is clear; therefore, to suit the physical difference of these two portions of the periphery, this text uses the trapezoidal formalization.

5.17. Control of discharge. For analysis an important feature of the canals was the restriction on discharge variations. Normally the laterals (para 5.9), from about 150 cusecs down, were run at full supply or were closed, so as to ensure fair distribution of water to the turnouts; more than 5% fluctuation was considered objectionable; however, some channels might be run for long periods at discharges deviating appreciably from the authorized. Before 1930 turnouts were mainly pipes or K-type outlets similar to the American Parshall Flume. These were replaced by stream-lined open flumes and by stream-lined orifices, both operating with hydraulic jumps so that discharge depended only on parent channel elevation and could be calculated theoretically. These devices were correct to about 3% and kept discharge distribution throughout each lateral closely to design. The theory of distribution has been recorded by Crump, 1922, who devised the new orifice—called an “adjustable proportional module (APM)”.

5.18. Branch discharges had to vary during short-supply seasons, but they were normally fitted with drops and gates to maintain head into the laterals. During short supply the ponding usually made the Branch beds inactive, so that low supply conditions were irrelevant to regime behaviour.

5.19. Generally, therefore, branches and laterals could be assumed to have equilibrium discharges close to designed full supply. How-
ever, some Branch and Main Line head reaches were in relatively
deep cutting in fairly inerodible soil and had been given slopes
steeper than for in-regime conditions. Some of these might not
be regime channels (para 4.2) although they had acquired per­
manent breadth, depth and slope at full supply discharge by eroding
in cohesive erodible material. They would be eliminated from
analysis.

5.20. **Control of sediment.** For regime analysis of a canal system
the important matter is that the individual channels should receive
sediment-load in proportion to their discharges; then the charge
is constant and will not disturb functional relations among the
other variables. Suspended load appears to distribute in the de­
sired manner automatically. Bed-load material has been known,
by irrigating farmers from time immemorial, to move to the in­
side of bends so that turnouts on the outside of bends are starved
of it. This phenomenon is responsible for bed-load dividing out
of proportion with discharge at offtakes, and this maldistribution
is with respect to quality as well as to quantity. However, in the
canals in question, bed-load charge in the river was very small
(as estimated from transport formulas based on, laboratory flumes)
and headworks design and regulation were aimed at making canal
bed-load charge less than that of the river; so charge had hardly
any effect on regime relations compared with size of bed-material.
The differentiation of coarse material into some offtakes, and fine
into others, was definite but was not associated markedly with
channel discharges throughout a whole canal system; this was be­
cause sediment is indestructible, so that if one offtake receives
more than proportionate share of coarser bed-material then the re­
mainder must receive less. So, within a system, a kind of natural
sediment control existed and channels could be regarded as re­
ceiving sediment fairly uniformly with random fluctuations from
this uniformity.

5.21. **Effect of suspended load on roughness.** From the start of
controlled canals every regulating engineer must have known that,
if his regulating gage was not influenced by a control point, then
he would have to reduce supply (according to the established gage­
discharge relation) by about 10% when water became very turbid
due to river flood; if he kept the gage constant about 10% extra
flow would pass. Effectively, this means that the immediate effect
of up to nearly 1% turbidity, mainly of material finer than 0.075
mm., is to reduce apparent roughness about 10%; as very high
turbidity would last only a few days, no definite statement can be
made about the effect on equilibrium values of regime quantities. Suspended load, not much more than just stated, also reduces apparent roughness in laboratory flumes with rigid boundaries and no bed-load material (Vanoni, 1946, Ansley, 1963).

5.22. **Log-normal distribution in bed-sand samples.** Literature indicates that the analysis of bed-sands for several decades satisfied canal engineers that there is some definite grain-size distribution law for bed-sands, but no attempt was made to discover its nature and the work of sedimentary petrographers appears to have been overlooked. The use of the $\phi$ system of measurement by the latter (Krumbein, 1938) indicates that they had found the Gaussian error function (normal) distribution in terms of the logarithm of particle size to be a good fit to facts as discussed already in para 3.7. It seems safe to assume the distribution applies to the canal sands that have partaken in dune-movement during an adequate period.

5.23. **Evidence of regime slope adjustment of rivers.** In connection with canal construction an investigation was reported by Foy (1944) on the effects of building barrages on the Indus system. The records led to the general conclusion that the inevitable sequel to raising river level at a barrage must be transmission of rise, slowly but surely, to the next effective control point upstream; the rise at the upstream point need not be the same as at the barrage since the introduction of canals alters equilibrium discharge and bed-load charge of the river. The finding from long-term records provides a check on the equilibrium concept (para 4.9) and is important because there is some engineering opinion that the effect of a reservoir on a regime river will not proceed upstream indefinitely unless the upstream load is enhanced by some means. Since Foy's report further river interferences seem to have prevented trends from becoming clearer (West Pakistan, Irrigation Research Institute, 1961).
CHAPTER 6

NATURE AND HISTORY OF FORMAL REGIME THEORY

6.1. Introduction. The application of regime theory philosophy and basic functional forms will occupy most of the remaining chapters of this text. It solves, to a useful degree of accuracy, most engineering problems of river evolution. This success has been possible because a definite philosophy has been followed, certain physically meaningful functional forms have been accepted as basic for specified simple physical conditions, and the existence of phases of flow-cum-transport outside the range of original observations has been kept in mind. The basic portion of the science has its own inductive foundation and does not rely on simplifications inherent in conventional engineering fluid mechanics of Newtonian fluids free from sediment and contained by immobile boundaries; the applied portion follows as logically as possible from the basic, does not pretend to be rigorous, but proceeds cautiously towards its objectives and emphasises deviations from rigor. As the essence of using such a subject is its clear physical understanding the present chapter will outline its development and nature as a preliminary and for future reference if difficulties arise.

6.2. Meaning of “regime theory”. The term “regime theory” has come to imply an inductive theory of channel self-formation that has grown systematically since about 1890, in terms of appreciation of canal self-adjustment and through measurements by engineers in the field (mainly in the Indian continent), has a quantitative orientation, and uses, where applicable, the functional forms of the equations of Lacey, 1929, 1933 in terms of directly measurable variables. The word “theory” is used in its dictionary senses of “a systematic conception or statement of the principles of something” or “an explanation based on observation and reasoning”; this usage applies to the classical physical theories of light, heat, electricity, etc., where speculation, which is another unfortunate meaning of the word, has no place and the basic equations came from observation.

6.3. Parallel with electromagnetism. The development of the science of electromagnetism was inductive and proceeded in ignorance of the nature of the peculiar intangible electric and magnetic
“fluids”. The line of attack was, generally, to correlate measurable variables, such as force and distance, associated with fixed “quantities” of these fluids. The coefficients in the correlations were then used to define measures of the quantities. The results were the equations of the electromagnetic field on which electrical engineering is based. The antithesis of electromagnetism is electronics, where the mechanism of electricity is studied and, necessarily, some speculation must be allowed; despite its advances it still cannot deduce the electromagnetic equations, but uses them. The two sciences are complementary. Basic regime theory obtained correlations among self-adjusted flow quantities of canals for conditions under which the exceedingly complex sediment conditions were approximately constant, and used the results to define a sediment parameter; thus the immediately hopeless task of understanding the mechanism of transport was relegated to the future. The antithesis of this approach is study of sediment transport, per se, in laboratory flumes. Both approaches are necessary for complete understanding and borrow from each other; however, each has a field of special utility in practice.

6.4. Early evolution of regime theory. Regime theory evolved slowly among engineers who started constructing modern-style irrigation canals in alluvium in the Indo-Gangetic Plain (Chapter 5) towards the end of last century. The breadths, depths and slopes that they designed were arbitrary except for an insufficient relation by the single flow formula of rigid boundary hydraulics. They became convinced, after decades of futile battling to make the channels retain these dimensions, of the general principle known also to geologists, that the provision of a supply of water and sediment fluctuating about long-term means must result in a channel whose self-formed dimensions also fluctuate about determinate long-term means. When a canal had won its battle it was said to have acquired regime, or to be “in regime”. The word “equilibrium” was also used, correctly, but did not become popular; its synonym is more descriptive of an obviously fluctuating pattern of behaviour controlled by less obvious but definite rules.

6.5. Lindley’s principle of channel self-adjustment. The principle quoted was published in presently acceptable form, to suit the canals in question, by E. S. Lindley, 1919; partially, it had received attention from R. G. Kennedy, 1895. The statement was: “When an artificial channel is used to convey silty water, both bed and banks scour or fill, changing depth, gradient, and width, until a
state of balance is attained at which the channel is said to be in regime,” The time required to formalise the principle may be compared with that needed to state the principle of the hydrologic cycle in a fuller form than presented by Ecclesiastes, “All the rivers run into the sea; yet the sea is not full; unto the place from where the rivers come, thither they return again.”

6.6. **Lacey analysis.** With the basic principle established, the next major advance was the collection and analysis, by Gerald Lacey, 1929, 1933, of a very wide range of wetted perimeters, hydraulic radii, slopes and discharges of canals believed to be in regime (para 4.4). Initial analysis was by individual canal systems so that the deviations of sediment conditions within any one system could be averaged out and the resulting equations would apply to the average conditions. The results permitted the definition of a “silt factor, f” for the system. Further analysis permitted f to be eliminated between equations so that new ones could be tested against channels of different systems provided only that they were in regime. The final result was three independent equations, one for each of the degrees of freedom (para 4.17) involved. These related the wetted perimeter, hydraulic radius and slope to discharge and to f. They were in general use throughout India within a few years and are still used in original or with adjustments that do not alter their functional forms in terms of measurable variables.

6.7. The quoted papers, their discussions, and personal contact, show that Lacey was aware of rigid-boundary fluid mechanics formulas and theories of the time and realised their unsuitability for application to a boundary so complex as the mobile one. Therefore his analysis followed the cautious inductive path of experimental physics. The work proceeded logically and ingeniously, step by step, seeking the most illuminating ways of relating data. Dimensional analysis was used on results, but not for preliminary statement of case. Power law fits to data were given simple exponents, instead of least squares ones, provided they were will within the limits of statistical significance; the reason was that all laws of physics are simple and, if of power law type, have simple indices. Dynamical incongruities were avoided. Possible relationship of the results to those of comparable hydraulic phenomena was discussed but appears to have had no influence on selection of the proposed equations.

6.8. **Generalization of Lacey formulas.** Work to date on the basic Lacey equations, by research institutes and individuals associated with canals of the Indian continent, may be viewed as resulting
mainly in expanding the silt-factor, f, to deal more fully with the properties of the water-sediment complex and with the difference in nature between bed and sides, or in the flow over them. N. K. Bose, 1939, proposed formulas in which f was replaced by mean bed-load size; in retrospect his work was an early indication that the indices of f in the original equations required examination. T. Blench, 1941, proposed a division of f into a bed-factor and side-factor to allow for the difference between cohesive erodible un-duned sides and non-cohesive duned bed, and indicated that the factors were actually implicit in the Lacey equations; he replaced wetted perimeter and hydraulic radius by breadth and depth. C. King, 1943, proposed a slope equation that allowed for the effect of breadth to depth ratio explicitly; in retrospect it can be seen as an amendment and expansion of f in a basic Lacey equation. Physically King's equation shows that the mobile boundary was a generalization of the "smooth" type of rigid boundary hydraulics and leads to interesting speculations on the nature of flow formulas in general. Inglis, 1949, emphasized the desirability of including bed-load charge explicitly in f and suggested a means deduced from dimensional analysis; he suggested, also, the practical use of different values of f in different formulas for any one canal. Blench and Erb, 1955(b), 1957, analysed the complete classic data of Gilbert, 1914, using regime theory parameters, showed that different formulas were needed for sub- and super-critical flows, drew attention to gaps in available data, and proposed crude empirical formulas relating bed-factor to bed-load charge of natural river bed sand; this converted King's equation to a sediment-transport one. Lacey 1958, expressed agreement to using breadth and depth in sectional equations instead of wetted perimeter and hydraulic radius, introduced a factor E into his slope equation, left f intact, agreed that charge was implicit in it, and appreciated the Inglis suggestion for its inclusion. Bhattacharya, 1960, and M. A. Qureshi, 1962, showed, in a laboratory flume, that the bed-factor depended on a channel parameter as well as on those of the water-sediment complex in the duneless subcritical flow condition that was not recorded by Gilbert and seems to occur in gravel rivers. Mushtaq Ahmed and Abdur Rehman, 1962, published observations showing the effect of suspended load on Lacey relations. Blench and Qureshi, 1964, proposed a design curve and rules for estimating, practically but roughly, in terms of medium size of bed-material of sand or gravel, the bed-factor for vanishingly small bed-load charge in subcritical flow; they produced a slope analysis chart
(Fig. 3.3) for rivers using a composite slope variable. An independent assessment of regime theory has been made by Thorn, 1966.

6.9. **Use of Gilbert data.** The Gilbert and Lacey extensive observations are remarkably complementary (Table 5.1). Lacey's were for relatively enormous ranges of breadth, depth, discharge and Reynolds Number, but for very little variation of bed-material size and side erodibility; the settlement law of the sand was in the anomalous range; bed-load charge was very small; flow was subcritical; the bed pattern was always duned. So the conditions were comparable with those of a laboratory where the undesirable variables could be held constant; hence, the information obtained about the major variables for one phase of flow was outstanding. Gilbert's were for relatively insignificant ranges of breadth, depth, Reynolds Number, and for moderate range of bed material size; sides were rigid but smooth; the settlement velocity law of the bed material ranged from anomalous to square root; bed-load charge range was large; flow was subcritical and supercritical; the bed showed duned, sheet and antidune conditions. So there were too many variables to permit the extraction of formulas accurately enough to approximate physical laws, but almost all the phases of flow-cum-transport that occur in rivers were present and identifiable visibly or by plotting results suitably. The combination of the two works is responsible for much of the quantitative application of regime theory formulas to rivers.

6.10. **Regime concepts in geology.** The engineering development of quantitative regime theory has been paralleled by geological appreciation of channel equilibrium coupled with quantitative observations that have been used and acknowledged in the engineering development. Inglis, 1949, quotes Jefferson, 1902, and Bates, 1939, on the linear average relation between river breadth and meander belt breadth, stating that the former permitted him to write a paper in which he developed a relationship between meander dimensions and a suitably defined discharge. Gilbert, 1914, conducted laboratory tests that are still, probably, the most exhaustive on record, aimed at investigating the causes of self-formation of depth and slope in rivers; his description of the dune, sheet and antidune phases of transport is quoted extensively in engineering and Blench and Erb, 1955(b), 1957, relied mainly on his data (as already stated) to convert the King regime slope equation into a transport one. Leopold and Maddock, 1953, made a broad regime theory type of analysis of breadth and depth data, by river systems. This has
produced information vital in testing the feasibility of extending canal regime equations, with due precautions, to rivers; it too, is quoted extensively in engineering literature.

6.11. Recent developments. Attempts, during the last decade, to extend beyond the limits of rigorous application of basic regime theory formulas into the phases shown by laboratory flumes, river models and smaller gravel rivers have started to show promising results since issue of the 1966 edition of this text. Reasons for recent rapid progress include (i) active cooperation, inspired by the International Hydrologic Decade, among workers with different viewpoints, (ii) growing appreciation of, and facility in the use of, dimensional analysis with physically correct statements of problems, (iii) the reduction, by computers, of the enormous task of analysing thousands of observations of quantities depending on four or five independent variables, (iv) increasing funds for education and research and (v) interest in the effects of engineering interference with river regime. The net result is cooperative interest in a new science of mobile-bed hydraulics embracing all phases of transport.

6.12. Improvement has grown from various directions. From the regime-theory viewpoint the analytic start may be seen in the adequate statement and dimensionless expression, in numerics suggested by regime-theory formulast, of a prototype bed-load transport experiment in a flume (Blench & Erb, 1957). The analysis of Gilbert data based on this, without computer aid, did not show the expected effect of relative particle size on bed-factor excess due to charge. However, Rottner, 1959, showed that the effect was predominant on bed-factor at vanishing charge. Yalin, 1965, produced a general treatise on dimensionless methods relevant to cases like the preceding prototype one and described a basic model experiment that appears to show how sediment specific gravity should enter any dynamically complete and correct formula. Kellerhals, 1963, 1967, broke away from the trifling range of particle sizes available in regime-theory canals and laboratory flumes by making a complete regime-theory type of analysis of gravel rivers and canals of very small charge; his data covered relative particles sizes larger than in flume experiments, and absolute sizes up to about 1 foot. He produced a complete set of regime equations for a phase different from that of sand-bed canals but closely linked to that of laboratory flume experiments that, in turn, just reach to the regime-theory canal phase. Cooper and Peterson, 1968, col-
lected world flume data and analysed, with computer aid, in terms of regime-theory based numerics for the two relevant degrees of freedom. The results confirm that, in general, changes of visible phase shown by the work of Simons and Richardson, 1966, are reflected in changes—more or less noticeable—in suitable plots. They show remediable defects in planning flume experiments, and major gaps that must be filled before formulas can be obtained to cover the field of engineering interest. The outstanding experimental advance has been in continuation and expansion of the Gilbert work, with modern facilities, by Simons, Richardson and Albertson, 1961, through Simons and Richardson, 1966. Visible bed-load transport phases have been described fully, mainly for sands, and measurements and methods have aimed to avoid older defects; field information and suspended load have received attention. Analyses have been performed in terms of a full statement of case and with a variety of numerics different from, but equivalent to, the regime-theory ones.

6.13. The present situation resembles the one that must have existed in rigid-boundary hydraulics when dimensional analysis of a statement of prototype case for flow in uniform circular pipes had just led to plots of data on a friction-factor diagram. Then disputes about “the proper flow formula” would start to be replaced by efforts to define phase limits and produce formulas for different phases; gaps in data would become glaring and lead to research in neglected areas; theorists could start to direct their speculations towards ascertained facts. Now a new science of mobile-bed hydraulics can embrace regime and other theories as partners. The friction-factor for bed-load transport in a flume is being recognized as multi-dimensional. It needs sheets of graphs for proper presentation, and their coordinates should be interchanged and manipulated in various ways to assist visualisation of the multi-dimensional geometry; this work is progressing and showing the gaps in data. The Froude Number (bed-factor, effectively) requires treatment similar to the friction-factor, to dispose of the second degree of freedom; this work too is progressing. Use of sonic sounders is leading to similar treatment for dimensionless bed-form wavelengths and amplitudes. Obviously there must be several years of improved fact-finding to complete the phase picture required by engineers and lay a foundation for the deductive side of the new science. Regime-theory is obviously competent to meet new demands in a cooperative effort.
CHAPTER 7

BASIC AUXILIARY AND DERIVED EQUATIONS FOR SAND-BED CANALS IN REGIME

7.1. Introduction. Chapter 4-6 are assumed to have been read so that equations required for use in future chapters can be presented now with minimum explanation. They will follow the sequence:

i. Three basic equations for the three degrees of freedom and the simple circumstances of idealised irrigation canals. The three dynamical statements they make will be included.

ii. The basic regime slope equation amended empirically to include the effect of bed-load charge.

iii. Three rough formulas and some instructions to assist in practical assessment of the bed and side factors, which are defined by the basic equations but are determined mainly by the natures of the bed and the sides.

iv. A set of formulas derived from the preceding ones for convenience in solving practical problems.

7.2. The basic equations here are not the original Lacey ones but, as outlined in Chapter 6, are generalizations resulting from various experiences since his presentation. When both sets are manipulated into relations among slope, discharge and representative depth and breadth they display exactly the same functional forms (as expressed by the indices of these quantities) and they make essentially the same dynamical statements; however, the Lacey equations define the nature of the water-sediment complex and of the sides by one “silt factor”, f, and make no explicit mention of bed-load charge whereas the equations of the text define a side factor as well as a bed factor and introduce bed-load charge explicitly into the slope equation. Fundamentally, therefore, this text accepts and uses the data and the functional forms of the Lacey equations in terms of directly measureable variables of the water flow.

7.3. Dynamical statements of the three basic regime equations. The three basic equations, with or without the empirical amendment (para 7.1(ii) ) to the slope one, can be shown to make the following three simple dynamical statements.

i. Channels with the same water-sediment complex tend to acquire the same Froude Number in terms of a suitable depth.
ii. The erosive attack on sides that behave as if hydraulically smooth can be measured in terms of the well-known criterion \( \frac{\rho \mu V^3}{b} \), where \( V \) is mean velocity of flow, \( \mu \) is dynamic viscosity, and \( b \) is a suitable breadth.

iii. Channels with the same water-sediment complex, and the same measure of erosive attack on sides, tend to adjust to the same dissipation of energy per unit mass per unit time.

The original Lacey equations would require omission, from iii, of the words “and the same measure of erosive attack on the sides.”

7.4. **Importance of the dynamical statements.** In engineering, observationally derived formulas seldom make simple dynamical statements that link with general indications from other knowledge so as to suggest dynamical laws; often they make none, and sometimes they state dynamical falsities. Therefore, and for other reasons, there seems a strong likelihood that the Lacey functional forms express dynamical laws related to energy dissipation, to transport capability, and to bank erosion for a particular phase (para 4.27) of flow-cum-transport. Obviously an engineer can build his experience more logically and efficiently if he can relate it to expectation from a simple but real dynamical framework; then also, as his knowledge grows, his chances of extending the framework improve.

7.5. **Conditions represented by basic equations.** The Lacey equations were obtained by smoothing the data of real canals to obtain behaviour of more ideal ones. Therefore they, and equations (7.1-3) should be held to apply rigorously only to real in-regime canals having the following major limitations:

i. Steady discharge.

ii. Steady bed-sediment discharge of too small an amount to appear explicitly in the equations.

iii. Duned sand bed with the particle size distribution natural in the sense of para 3.7.

iv. Suspended load insufficient to affect the equations.

v. Steep cohesive sides that are erodible or depositable from suspension and behave as hydraulically smooth.

vi. Straightness in plan, so that the smoothed duned bed is level across the cross-section.

vii. Uniform section and slope.

viii. Constant water viscosity.

ix. Range of important parameters as in column 1, Table 5.1, or in whatever extrapolated range permits the same phase of flow.
(Specifically, the equations are unlikely to apply if b/d falls below about 5, or depth below about 400 millimeters.) Practically the equations would apply satisfactorily, at authorized full supply discharge, to most well-maintained irrigation canal systems with sand beds.

7.6. The basic equations for trifing bed-load charge. The equations accepted as basic in this text are:

\[ \frac{V^2}{d} = F_b \]  \hspace{1cm} (7.1)

\[ \frac{V^3}{b} = F_s \]  \hspace{1cm} (7.2)

\[ \frac{V^2}{gdS} = 3.63 \left( \frac{Vb}{\nu} \right)^{1/4} \]  \hspace{1cm} (7.3)

and must be read in the context of paras 7.4 and 7.5. The terms will be held to mean as follows: V is the mean velocity of flow; S is the energy gradient which equals the channel slope; \( \nu \) is the kinematic viscosity of the water; d is the depth from water-surface to smoothed (in imagination) duned bed; b is the mean breadth given by bd = A where A is the cross-sectional area of the flow. Definition of b and d is needed for practical purposes but the perfect definition for basic purposes is not known; hence the use of the terms “suitable depth” and “suitable breadth” in para 7.3. If surface breadth were used for b, and bd still remained equal to A, so as to redefine d, then the coefficient 3.63 in equation (7.3) would need a couple of percent adjustment since the sides are assumed steep (para 7.5, v) and b/d is assumed fairly large (para 7.5 (ix)).

7.7. Discussion of the basic equations. Equation (7.1) defines a bed-factor which, in practice, depends mainly on the nature and the charge of the bed-material. Dividing both sides by \( \nu \) converts the left-hand side into a Froude Number and explains the origin of the first dynamical statement of para 7.3. Equation (7.2) defines a side-factor which, in practice, can lie between limits depending on the erodibility of the sides, the viscosity of the fluid, and the tendency for suspended material to deposit and form sides. Multiplying both sides by \( \mu_\rho \) leads to the second dynamical statement of para 7.3. Equation (7.3) is the flow formula and is of the same functional form as the Blasius one for rigid boundary and Newtonian fluids when boundary conditions are “smooth”, which means that the boundary resistance arises from a layer of the fluid in a special phase; so it reflects the fact that the mobile boundary consists of a differentiated portion of the water-sediment complex. Algebraic manipulation of all three equations yields:

\[ gV^2S = F_b \cdot \left( \frac{\nu F_s}{\nu^2} \right)^{1/4} \]  \hspace{1cm} (7.4)

53
and is the origin of the third dynamical statement of para 7.3. The first two equations are dimensional and were framed for feet and second units; introduction of $g, u, \rho$ into them would have made their dynamical statements obvious, but would have burdened all practical calculations in which they were used without adding anything to their utility. The third equation is in a standard nondimensional form used regularly by engineers in connection with “friction factor” in rigid-boundary fluid mechanics.

7.8. The flow-formula adjusted for appreciable bed-load charge. Bed-load charge was inadequate, in the Lacey data on which the three basic equations were based, to show any need for inclusion along with the overall “silt factor”, $f$, or along with $F_b$ which replaces it in the equations of this text. Under world conditions bed-load is likely to remain unimportant in irrigation distributing canals as a whole although exceptions can be imagined; in link canals between rivers the chances of significant bed-load are higher; in canals used to start cutoffs in rivers significant bed-load charges are essential if the cutoff is to develop. Blench and Erb (para 6.10) made a rough practical amendment to equation (7.3), in terms mainly of Gilbert data, and recommended:

$$V^2/gdS = 3.63\left(1 + C/233\right)\left(V_b/v\right)^{1/4}$$

... (7.3a)

for subcritical flow only and with the understanding that present data did not justify anything more refined. Here $C$ is the bed-load charge (para 4.25) in parts per 100,000. This formula will replace equation (7.3) for all practical work. It is not basic, since there is no obvious dynamical significance in the factor $(1+C/233)$, and other forms of the factor could be justified from the data within the limits of statistical significance. As $C$ tends to zero the formula tends to the basic form of equation (7.3). It alters equation (7.4) to:

$$gVS = F_b \cdot \left(vF_s\right)^{1/4}/\left[3.63\left(1 + C/233\right)\right]$$

... (7.4a)

so does not alter the wording of the third dynamical statement of para 7.3. Equation (7.3a) might be called “near basic”. The Lacey equivalent of equation (7.4a) is $gVS = 0.014\, f^2$ in foot second units.

7.9. Status of rough formulas for bed and side factors. In status and use there is a close parallel between bed and side factor in mobile boundary work on the one hand, and roughness factor or its alternative, the “roughness height” (which is a roughness factor adjusted to have the dimension of length), in rigid boundary fluid mechanics on the other hand. In practice all these factors are determined by a multiplicity of sub-factors (e.g. grain size, shape, spac-
that contain some unmeasurable and undefinable items; so no accurate quantitative inter-relation can be devised by either measurement or theory. The factors have to be defined, therefore, by accepting a formula obtained, by observation or by speculation, as a relation among measurable variables of the flow under conditions which are believed to make the factors constant; the coefficient in such a correlation is used for the definition.

7.10. Although a crude formula for determining such a factor is a convenience, experience shows that the engineer can manage very well, for the case of the roughness factor, without any formula at all. He inserts, for actual cases with different boundaries, measured variables in his preferred flow formula, computes the values of the roughness factors and remembers them in association with the general boundary appearances. For a few special boundaries he can devise crude formulas to express his observations. For example there is a rough rule that, for immobile gravel boundaries, Manning's $n$ is about one fortyeth of the sixth root of the median gravel size in inches.

7.11. For the bed-factor the formula situation is a little better than for a roughness factor applicable to a rigid boundary. The median size of a bed-sand can be measured readily and the existence of a standard natural size distribution in samples makes the specification of distribution unnecessary; grains, too, have fairly standard shape, and specific gravity. Virtually, therefore, one measurable quantity defines the mobile sand bed, whereas a rough rigid boundary would usually require measurement of a half-dozen quantities of which most are totally unmeasurable. However, bed-factor depends on bed-load charge, as well as on material size, and charge cannot be measured successfully as a routine in the field. The formulas available to aid the engineer in preliminary assessment of bed-factor are given below; their status is comparable with that of the Manning's $n$ rule for gravel in para 7.10. However, the best way to learn about bed-factor values remains that of inserting measured quantities in suitable regime formulas that contain $F_b$, and classifying and remembering the circumstances under which the values of $F_b$ were found; paras 8.6, 11 illustrate.

7.12. **Rough formula between bed-factor and bed-load charge.** The necessarily rough formula to be used for expressing bed-factor in terms of bed-load charge less than about 10 is:

$$F_b = F_{bo} (1 + 0.12C) \quad (7.5)$$
where \( C \) is the bed-load charge of natural sand in parts per hundred thousand by weight of the fluid discharge (para 4.26). Its form results from analysis by Blench and Erb of Gilbert data (para 6.10) for rather unnatural bed material distribution; the value 0.12 is proposed to suit limited information from natural sands. It provides the simplest fit available to scattered data within the limits of statistical significance, and does not apply to super-critical flow. For super-critical flow, with the unnatural Gilbert bed materials, a formula \( F_b = 32.2 + 0.06(C - C_c) \) was found, where \( C_c \) is the charge corresponding to critical velocity—that is, to \( F_b = 32.2 \); it will not be used here.

7.13. **Definition of “zero bed-factor”, \( F_{b0} \).** Obviously \( F_{b0} \) in equation (7.5) is the value of \( F_b \) when \( C \) is vanishingly small, so the \( F_b \) in the basic regime equations is really \( F_{b0} \) (para 7.5, ii). It is called the “zero bed-factor”, meaning the value to which the bed-factor tends as \( C \) tends to zero. Strictly a bed-factor cannot be defined for \( C \) equal to zero since the channel is not then a regime one according to para 1.1; the qualification that zero had been reached, without loss of dune pattern, by reduction from a defined value would be required to make the definition legitimate.

7.14. **Rough relation of zero bed-factor to bed-material.** For the conditions to which the basic regime equations apply (para 7.5), and for practical sand-bed canal conditions generally, \( F_{b0} \) depends principally on mean bed-particle size. The formula normally used is:

\[
F_{b0} = 1.9\sqrt{D_{mm}} \tag{7.6}
\]

where \( D_{mm} \) is the median bed-material size by weight in millimeters; that is, it is the size that is exceeded by half the weight of a large bed-sample in which the particles have been arranged in order of size. Equation (7.6) is equivalent to the “very rough qualitative formula” proposed by Lacey, 1930, in the form of \( D \) inches = \( f^2/64 \), which is still in use. The sand-size portion of Fig. 7.3 is believed to be a refinement replacing Eq (7.6); for its gravel portion see para 10.44.

7.15. **Systems of measuring particle sizes.** The sizes of individual sand particles, from which \( D \) is to be calculated, will usually be available in terms of one of three different systems—not always specified in texts. The measures used in these systems are:

i. **Spherical or nominal diameter.** The diameter of a sphere of the same volume as the particle.

ii. **Standard fall, or standard settlement diameter.** The diameter
of a sphere, of specific gravity 2.65, that would have the same terminal velocity as the particle in infinite still pure water at 24 degrees Centigrade.

iii. **Sieve diameter.** The length of the side of the smallest square opening through which the particle will pass.

The relation among these different measures is given in Table 7.1, prepared from graphs given by Interagency Committee on Water Resources, 1957, which deals with the many refinements of sediment measurement. The “shape factor” of the Table, explained in the reference, has the value 0.7 believed suitable for sand.

**Table 7.1. Sand particle diameters in mm. at 0.7 shape factor.**

<table>
<thead>
<tr>
<th></th>
<th>.071</th>
<th>.11</th>
<th>.22</th>
<th>.34</th>
<th>.60</th>
<th>.74</th>
<th>1.35</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spherical or nominal</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Standard fall or settlement</td>
<td>.07</td>
<td>.1</td>
<td>.2</td>
<td>.3</td>
<td>.5</td>
<td>.6</td>
<td>1.0</td>
</tr>
<tr>
<td>Sieve</td>
<td>.063</td>
<td>.092</td>
<td>.2</td>
<td>.3</td>
<td>.54</td>
<td>.67</td>
<td>1.35</td>
</tr>
</tbody>
</table>

Most sands will have sizes between 0.1 and 0.6 mm., with the preponderance towards the middle of the range, so the question of which system to use in a rough formula like equation (7.6) is academic. Formally, however, the equation will be taken to hold for median standard fall diameter, since sand size determination by settlement tube is common and convenient.

**7.16. Phase, and indications of dimensional analysis.** It can be shown from dimensional analysis that non-dimensional bed-factor $F_b/g$ should be expressible, in general, in terms of:

$$C, \left( \frac{\nu g}{v} \right)^{1/3} \frac{D}{v}, \frac{b}{d}, \frac{d}{D}, s \quad \text{...........................................(7.7)}$$

where $D$ is some mean particle size and $s$ is the density of the bed solids relative to the fluid. To these must be added a variety of non-dimensional parameters to deal with size distribution, shape and shape distribution of the bed-material, its specific gravity distribution among particles if relevant, and finally all the parameters of the suspended load. Neglecting all these, and $s$, as practically constant, or irrelevant for the conditions of the basic equations, the fact remains that $b/d$ and $d/D$ have been omitted from the formulas (7.5) and (7.6). The warning that they can be omitted only within certain limits is given in para 7.5 (ix); for small sand-bed models they should be expected to be significant. Chapter 10 gives details.

**7.17. Alternative rough formula for $F_{bo}$.** An alternative, adapted from Blench and Qureshi 1964, to equation (7.6) is:

$$F_{bo} = 0.58V_s^{11/24} \left( \frac{\nu_0/\nu}{\gamma} \right)^{11/72} \quad \text{..............(7.6a)}$$
where \( V_s \) is the terminal velocity of settlement of the median bed sand size in the water of the canal at 70°F.; \( \nu_{70} \) is the kinematic viscosity of the water at 70°F. The curious indices arise from trying to devise an equation to fit gravel, for \( d/D \) large enough, as well as sand, allow for the peculiar shapes of gravels, and not suggest a basic formula by means of simple indices; equation (7.6) does not apply to coarse gravel at all. There is a possibility that the indices should be 1/2 and 1/6; the equation can be made dimensionally correct by introducing \( g \) suitably. The formula has some merit as a speculative starting form for \( F_{in} \) for all sizes of material, to be revised when research provides greatly improved data, and for research problems such as whether temperature affects bed-load transport; for practical purposes equation (7.6) seems preferable, at least in the intermediate sand range. Both equations rest mainly on rather indirect field experience, and on reasonably successful applications to special problems. Figs. 7.1, 3 are provided for use with equation (7.6a).

7.18. **Practical values of side-factor \( F_s \).** According to the second dynamical statement of para 7.3, \( \rho \mu F_s \) measures erosive attack on sides. If the attack exceeds some limit then the sides will erode till the attack drops to what they can withstand. If the attack falls short of a lower limit then materials from the total load can deposit on the sides, and the channel breadth will decrease. If the attack is between these limits nothing will happen to the sides. So \( \rho \mu F_s \) is not a single-valued quantity determined by parameters of the water-sediment complex and the bank material; any value between certain limits can be imposed on a particular canal and it will run in-regime at the imposed value as long as the other imposed conditions remain fixed. The nature of erosive resistance of cohesive banks under natural conditions, and of depositability of suspended load, are too complex for any formulas to have been devised to fix limits of \( \rho \mu F_s \). A rough guide to the upper limit is that very sandy loam banks are likely to erode with \( F_s \text{ft}^2/\text{sec}^3 \) greater than about 0.1; silty clay loam may yield around \( F_s = 0.20 \); not many very cohesive banks will stand indefinitely against \( F_s = 0.3 \) however well they may wear for a couple of years. With considerable suspended load, channel breadth may fluctuate seasonally in a real channel.

7.19. When sides are made of bed-material, \( F_s \) and \( F_b \) are related and rough formulas can be proposed linking them. This state of affairs is improbable in an irrigation canal with suspended load, but will be dealt with in discussing gravel rivers.
7.20. Derived equations for practical use. The engineer and the designer think of an in-regime canal, to a first approximation, in terms of the ideal conditions of para 7.5, using an equilibrium discharge and assuming the possibility of a small but significant bed-load charge. This is reasonable since the basic equations derive from canal conditions. Their interest is in the dimensions $b, d, S$ that will form as a consequence of imposing a full supply discharge, $Q$, a bed-factor $F_b$ associated with a charge $C$, and conditions that will limit $F_s$ within a range. The basic equations are in an awkward form for finding $b, d, S$ from their physical causes, so the following practical equations are derived algebraically.

7.21. Sectional equations. Solving equations (7.1, 2):

$$b = \sqrt{F_b Q / F_s} \quad \text{(7.8)}$$

$$d = \sqrt[3]{F_b Q / F_s^2} \quad \text{(7.9)}$$

These are called the “sectional equations” as they do not contain slope. It is interesting to note that they are algebraic identities, true even for concrete channels; they acquire special meaning in regime theory merely because nature compels a definite value for $F_b$ in any one problem and prescribes a range of values for $F_s$ if sides are self-adjusting.

7.22. Slope equations. Solving equations (7.1, 2, 3a) the alternative slope formulas are obtained:

$$S = \frac{[F_b^{5/6} F_s^{1/12}] / [K Q^{1/3} (1+C/233)]}{(7.10')}$$

$$S = \frac{[F_b^{7/8}] / [K b^{1/12} d^{1/8} (1+C/233)]}{(7.10'')}$$

$$S = \frac{[F_b^{11/12}] / [K b^{1/6} Q^{1/12} (1+C/233)]}{(7.10''')}$$

where $K = 3.63 g v^{1/4}$. The equations become basic as $C$ tends to zero. It is convenient to rewrite these equations to contain $F_{bo}$, which depends only on $D$. Replacing $F_b$ according to equation (7.5), for $C$ less than about 10, gives:

$$S = \frac{[F_{bo}^{5/6} F_s^{1/12} / K Q^{1/6}]}{f'(C)} \quad \text{(7.11')},$$

$$S = \frac{[F_{bo}^{7/8} / K b^{1/6} d^{1/8}]}{f''(C)} \quad \text{(7.11'')}$$

$$S = \frac{[F_{bo}^{11/12} / K b^{1/6} Q^{1/12}]}{f'''(C)} \quad \text{(7.11''''})$$

where the expressions for the fns of $C$ are written on Fig. 7.2 which has been prepared to avoid computing the functions.

7.23. Miscellaneous equations. A useful formula when breadth is known, so that $F_s$ becomes irrelevant, is:

$$d = \left(\frac{q^2}{F_b}\right)^{1/3} \quad \text{(7.12)}$$
For research purposes, and improved physical understanding, the flow formula (7.3a) can be rewritten as:

\[ V = (d/x)^{1/4} (gdS)^{1/2} \tag{7.3b} \]

where the equivalent roughness height is:

\[ x = (vF_s)^{1/2} / [3.63^2F_b (1+C/233)^2] \tag{7.3c} \]

The velocity formula:

\[ V = (F_bF_sQ)^{1/a} \tag{7.13} \]

is useful in model scaling.

N.B. Important formulas are collected in Appendix 2. In metric units 3.63 and 233, being dimensionless, are unaltered.
CHAPTER 8

FORMULA APPLICATION. CANALS OF SMALL BED-LOAD CHARGE OF SAND: DISCHARGE EFFECTIVELY STEADY

8.1. Introduction. The remainder of this text will make maximum use of the formulas of Chapter 7 for solving engineering problems and developing physical understanding since other phases are poorly formulated and less important. The procedure will be to start with simple canal systems to which the basic equations of para 7.6 can be applied almost exactly and then remove restrictions on steadiness of discharge, magnitude of bed-load, and finally meandering till the canals have developed into rivers. Worked problems will be used extensively to illustrate methods and to illustrate points of river behaviour that will arise later; they will all contain some practical lesson. The present chapter will be confined to canals similar to those from which the basic equations were obtained, so that, at least as a first approximation, those equations may be assumed to apply exactly; the equilibrium discharge (para 4.14) used in them will be an authorized full supply because the beds are assumed to become inactive at slightly less discharge under practical running conditions and because appreciably larger discharges are not permitted to flow. The exact conditions are in para 7.5.

8.2. Outline behaviour of the real canal system. No canal system or single channel has the ideal steadiness or regularity hypothesized for the basic equations. The parent river suffers secular changes of water-sediment composition due to climatic cycles and human interferences with ground cover that affect the source of sediment. The canal system draws a sediment load whose quality and charge are affected by these secular changes, by natural changes of orientation of the river at the barrage, and by engineered changes of approach conditions and regulation at the canal head. Within the system no bed-load enters the Main Line when the river is low, so the channels near the head draw on their bed-material during such periods and degrade in consequence; channels well downstream show little or nothing of this effect. Bars of coarse sediment may pass through the system at about a mile a year because of happenings such as the river breaching into the Main Line in high flood, or the engi-
neering practice of dumping sediment-laden cross-drainages into the system. Repeated engineering actions such as raising or lowering drops, altering head-regulators, sediment clearing, and changing capacities all produce a sequence of secular changes remote from the immediate disturbances. So the various factors observed in the in-regime condition possess a kind of rhythm, rather like that of the body processes of a person leading a normal active life and retaining his “constant” good health. The rhythm is not very obvious to superficial observation, but becomes apparent in applying the equations for ideal conditions.

8.3. **Trend of dimensions within a canal system.** For project design of a system a decision would be needed on (i) the probable time-average bed-factor that would be imposed, eventually, by nature aided by the engineers and (ii) a designed side-factor within the range nature would permit. Then nature would be anticipated by designing b, d, S to suit, for the equilibrium discharges of the various canals of the system. A tabulation of designed dimensions shows certain trends vividly.

**Problem 8.1.** For project design it is assumed that every canal of a system will have the same bed-factor and the same side-factor when in-regime. Experience indicates that nature, aided by the engineers, will impose \( F_b = 1.0 \) and permit \( F_s = 0.20 \). \( C/233 \) in equation (7.3a) is assumed negligible and \( K \) is taken as 2,000 to suit the average yearly temperature of the canal water. The equilibrium discharge, \( Q \), is assumed to be the authorized full supply and to be usable in the regime equations to determine \( b, d, S \). Tabulate for \( Q = 5,320 \) and \( 20,480 \) cusecs, noting that these discharges form a geometric progression with ratio = the sixth power of 2.

**Answer.** Use equations (7.8, 9, 10'); \( S_* \) means \( S \times 10^3 \) which is slope per thousand; columns 7-9 are for later discussion.

<table>
<thead>
<tr>
<th>( Q ) (cusecs)</th>
<th>( b ) (ft.)</th>
<th>( d ) (ft.)</th>
<th>( S_* )</th>
<th>( b/d )</th>
<th>( V )</th>
<th>( V_S* )</th>
<th>( dS_* )</th>
<th>( dV_S* )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
<td>(5)</td>
<td>(6)</td>
<td>(7)</td>
<td>(8)</td>
<td>(9)</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>1</td>
<td>.335</td>
<td>5</td>
<td>1</td>
<td>.335</td>
<td>.335</td>
<td>.335</td>
</tr>
<tr>
<td>320</td>
<td>40</td>
<td>4</td>
<td>.1675</td>
<td>10</td>
<td>2</td>
<td>.335</td>
<td>.670</td>
<td>1.340</td>
</tr>
<tr>
<td>20,480</td>
<td>320</td>
<td>16</td>
<td>.0838</td>
<td>20</td>
<td>4</td>
<td>.335</td>
<td>1.340</td>
<td>5.360</td>
</tr>
</tbody>
</table>

The trend shown by columns (1-6) of the answer to question 8.1 is that, relative to small channels, large channels in their equilibrium condition run broad, deep, flat, shallow relative to breadth,
and fast. The same trend is observed among river channels of comparable water-sediment complexes despite the difficulty of deciding suitable measures for some of the dimensions (Chapter 3).

8.4 Test of certain non-silting non-scouring hypotheses. Common engineering hypotheses about “non-silting non-scouring” (that is, in-regime) channels of the same watersediment complex are that they possess constant (i) velocity, (ii) “tractive force”, which is practically $\rho g dS$, (iii) energy dissipation per unit time per unit perimeter, which is $\rho g dVS$. Columns (6, 8, 9) of the table of Problem 8.1 show that the hypotheses do not hold in the range of practical irrigation canals. Column (7) checks the constancy of the dissipation of energy per unit mass per unit time.

8.5. Unsuitability of first basic equation for finding $F_b$. Paras 7.9-11 discussed the need for finding bed-factors from the running conditions of existing canals. At first sight basic equation (7.1) might seem the obvious one to use for the purpose in conjunction with regular discharge observations. A mathematical objection is that the formula is very sensitive to a variety of random and biased errors, including a poor guess at the equilibrium discharge. There are strong practical objections also. A major one is that the accurate design of a canal’s regime slope is most important for water distribution to offtakes, whereas design of sections can be relatively inexact without any serious practical consequences; if $F_b$ is estimated wrongly, because of wrong observations by the $V, d$ method, the cost of later maintenance of slope will be unnecessarily heavy. Again, one discharge at one section is of little use; several discharges over a considerable period at several sections are needed.

**Problem 8.2.** What are the approx. errors or deviations in the observed value of $V^2/d$ under the following separate circumstances: (i) data are used from a discharge observation in which the current meter recorded 3% below truth due to turbulence and depths were recorded 5% too much due to using a sounding rod without a baseplate, (ii) the true data are used from a short turbidity period during which 6% more discharge ran, at a given gage, than normally, (iii) the discharge is taken correctly from a long-crested weir and the depth is measured exactly but, because of passage of a short sand bar, is 5% less than normal.

**Answer.** (i) Approx. error $2 \times (−3) + (−1) \times (5) = −11\%$ (ii) Deviation $2 \times 6 = 12\%$. (iii) Because $V^2/d = Q^2/(b^2d^3)$, the deviation is $−3 \times (−5) = 15\%$ approx.
8.6. **Advantages of third basic equation for finding \( F_b \)** The basic slope equation reduced to the form of equation (7.10'') is tremendously effective. To use it to find \( F_b \), the slope should be measured in a long reach free from backwater. This slope, being the same over a wide range of discharges and free from short period change, is the correct equilibrium slope, subject only to secular change. It measures virtually a time-space average over a length of canal, whereas one \( V^2/d \) value is an instantaneous point value. It can be measured practically to 1% or better. In the formula the sixth root for \( b \) eliminates practical error on account of \( b \), and the twelfth root for \( Q \) has a similar effect—in fact, estimation of \( Q \) from standard annual hydraulic survey data would probably suffice for most calculations. Even \( \nu \), which should be an annual mean since regime formulas concern equilibrium values—not instantaneous—occurs to the fourth root, so cannot produce appreciable error. In the canals considered \( C/233 \) can be neglected; even in others where it could not then \( (1+C/233) \) would be included in computed \( F_b \) and, in using this \( F_b \) to design new canals of the observed system, the error would cancel out while designing \( S \) unless the new canals had a different \( C \).

**Problem 8.3.** What are the numerical % errors in \( F_b \) due separately to the numerical % errors noted against each of the following: \( S \), 1.1%; \( b \), 1.1%; \( Q \), 11%; the average annual \( \nu \), 1.1%.

**Answer.** Since \( F_b^{11} = S^{12} \cdot b^{0.2} \cdot Q \cdot \nu^{-3} \) the %s are 1.2; 0.2; 1.0; 0.30.

A question sometimes raised is how the equation appears to be insensitive even to errors of bias such as demonstrated in Problem 8.2. Actually, the coefficient 3.63 contains a kind of averaged-out bias of all the observations on which it is based, so is appropriate to the standard ways of conducting observations; the user is spared from conducting special sets of observations affected by his own particular non-standard bias. If methods of approximating the true discharges and depths of canals become improved from their present standard the coefficient may require alteration.

8.7. **Distribution of \( F_b \) in a system.** An engineer taking charge of a canal system would be wise to estimate \( F_b \), from equation (7.10''), for every channel head reach, and for reaches just upstream and just downstream of every offtake. The necessary data would normally be on record. The results would be marked on a key diagram of the system for study. If every reach were in-regime then normal expectation is that offtake bed-factors would differ appreciably from parent values upstream, and the parent values
downstream would differ in the opposite direction. This is because bed-sediment does not normally distribute uniformly in quality or in proportion to discharge, and the total quantity of sediment cannot be changed. Deviations of more than 10% at an offtake merit investigation. Causes are various. An offtake that is sediment-cleared regularly in its head will show an abnormally small value of $F_b$ because it is not allowed to attain in-regime slope. A large channel that shows a remarkably high $F_b$ may be in cutting in relatively inerodible soil that has never been able to flatten to a slope that permits a fully developed sand bed at authorised full supply; then it is not a regime channel and the regime formulas do not apply. Its state can be checked from old surveys, from inspection during closure (being careful not to be misled by sand pockets, since an eroded cohesive bed is often irregular), and from sonic surveys for dunes during full-stage running. If a system is fitted throughout with deep-set undershot head gates, then every offtake may have a larger bed-factor than its parent upstream, and bed-factors will decrease steadily from head to tail of the system. Because of differentiation arising from centrifugal forces, channels taking off from the concave side of a bend will tend to have appreciably smaller bed-factors than ones of comparable size and the same type head-regulator on the convex side of a bend.

8.8. **Value of $F_b$ distribution study.** After a quantitative study, as described, the engineer will know all the channels that need sediment exclusion and all the ones that can take more sediment without drowning out their heads. He will soon find what types of heads and excluders or collectors are efficient, and be able to estimate the approximate amount by which they can alter $F_b$ of the offtakes they control. Automatically he will begin to acquire the ability to estimate likely $F_b$ values for new channels rather like a pipe engineer learns to guess roughness factors.

8.9. **Sediment differentiation in rivers.** The marked differentiation of sediment size distribution across a curved channel section, as corroborated by bed-factor analyses, must occur in rivers. It can explain certain peculiarities of river regime. For example, where a river splits at a delta there may be, for some time, a condition where the water-sediment complex of each arm is just correct for in-regime conditions with the imposed discharge cycle. If meandering then alters the approach conditions to the bifurcation, the division of complexes will change so that the arm on the outside of the curved approach will start to adjust to suit a smaller bed-
factor; therefore it will erode and take more discharge and, therefore, erode even more because its regime slope will have become still less than its imposed slope. The tendency will be for the growing channel to capture the whole river, while the other “dies”; however, as the shrinking channel becomes smaller its bed will rise and act rather like a high head-regulator sill and thereby exclude coarser sediment so that, eventually, a small enough bed-factor may be acquired for the shrinking to stop. The complete behaviour will depend on the nature of the complex and the nature and rate of development of the meander cycle, if not on other factors.

8.10. Side-factors of a single canal reach. Determination of the side-factor of a reach is of limited practical use by itself, for the value found is only one in the range of possibilities for the given conditions (para 7.18), and the top of the range may vary from place to place due to changed bank conditions. The general objections (para 8.5) to using equation (7.1) to find $F_b$ apply even more strongly to using equation (7.2) to find $F_s$. A fairly insensitive formula is:

$$F_s^{7/12} = K(dS/b^{1/3})(1+C/233)$$ ..............................(8.1)

which is found by eliminating $F_b$ and $Q$ from equations (7.8, 9, 10”). The value obtained is for the accepted equilibrium depth, $d$.

8.11. Mean bed and side factor for a whole system. For the record of a project designer a collection showing the average bed-factor and side-factor for each of several systems is valuable. The best average for a system is obtained from individual values for its in-regime channels, using equations (7.10””, 8.1); whether to use an unweighted average, or to weight in terms of $Q$ or some other variable, is debatable, and there is probably little practical difference among the alternatives. However, the use of equations (7.1, 2) is not as objectionable as for a single channel reach, since random errors will tend to average out, but still leaves wrong choice of equilibrium condition effective along with other sources of bias. To use equations (7.1, 2) plots would be made of $V$ against $d$ and $b$ separately and fitted, on double-log paper, with lines sloping at 1/2 and 1/3 respectively; if the points showed a definite bias off these slopes a field investigation would be merited, to discover the physical causes. An alternative, leading to the same result, is to plot $d$ and $b$ separately against $Q$.

Problem 8.4. Extensive field research data of a long period
for a system of channels between 15 and 750 cusecs gave, at
designed full supply discharge:

\[ b = 2.505Q^{1/2}, \quad d = 0.503Q^{1/3} \]

Estimate the overall bed and side factors of the system for the
specified discharge level.

**Answer.** \( F_b/F_s = 2.505^2; \quad F_s/F_b^2 = 0.503^2 \).
Therefore \( F_b = 1.25, \quad F_s = 0.20 \).

8.12. **Testing for in-regime conditions.** Frequently, in practice,
one is provided with data from a channel alleged to be in regime,
running at its equilibrium discharge, and required to use this
information for making a decision. Caution is required since mis­
takes in assessing in-regime conditions and in collecting proper
data are common. For example, a canal cut into cohesive soil may
have achieved a steady condition without having flattened enough
to have a genuine non-cohesive bed; its steadiness would then not
indicate an in-regime conditions for applying regime conditions
at part supply, or shortly after the removal of a drop structure had
upset regime. In systems that have not been under good engineer­
ing control the survey data may be wrong. If insertion of alleged
regime data into independent regime equations and consideration
of the results lead to inconsistencies or inexplicable peculiarities
the data should not be used for a decision until the causes have
been discovered.

**Problem 8.5.** The following data were quoted, for one channel
reach, in a list from a system known to be generally of regime
type. \( Q = 150 \) cusecs, \( D_{mm} = 0.32 \text{mm.}, \quad r = 1.0 \times 10^{-5}, \)
bank material very cohesive, \( d = 3.5 \text{ ft.}, \quad b = 31 \text{ ft.}, \quad S_{\alpha} = 0.135. \) Were
they for in-regime conditions? If not, comment.

**Answer.** \( V = 1.38 \) ft/sec., \( V^2/d = 0.54 \). But \( F_{bo} \) approx. =

\[ 1.9\sqrt{0.32} = 1.07. \] Therefore, from equation (7.5), charge \( C \) =
minus 4 approx. As a minus value is impossible physically, it
appears that the channel is running at conditions that could not
make the bed move. Further, taking \( K \) as 2,000 to suit the value
of \( r \), equation (7.10”) gives \( S_{\alpha} = 0.20 \) with \( F_b = F_{bo} \). The actual
slope of 0.135 per thousand is so much less than this that bed­
movement would be impossible. So the channel is not in-regime.
A further check on actual conditions is \( V^2/b = 0.085. \) A designer
would hardly design to this for material that would stand quite
0.25 (para 7.18), for it would give very broad channels; so it
seems that the channel was running low supply or was ponded
up. The observer would probably have been aware of low sup-
ply, so the chances are that the channel was running ponded
up. Manning’s $n$ works out at 0.024 which is high enough to
suggest that the bed had not lost the dunes that would be mov-
ing in the unponded state.

8.13. **A special lined canal problem.** The following problem illus-
trates an occurrence in a lined canal that was built unusually
shallow because of waterlogging. Normally, lined canals are built
with bed and sides tangent to a semicircle so as to reduce lining to
a minimum; this shape reduces the likelihood of sediment deposition
during part-supply but the relative importance of sides then calls
for some caution in applying the regime equations. The general
finding is that, if a lined canal is liable to receive a deposit of bed-
sediment during abnormal operating conditions, it will need to
have at least the regime slope of an unlined canal if it is to be
self-cleansing.

**Problem 8.6.** A lined canal, Manning’s $n = 0.015$, was built with
bed-width 140 ft., side slopes 1:1, slope 0.09/1,000, to take 6,045
cusecs at 10 ft. deep. The water carried sediment load that, in
a regime channel, would have resulted in $F_{bh} = 1.0$ though
$C/233$ was negligible; $K$ would be about 2,000. The canal ran
successfully for a couple of years as a link between two rivers.
Later, when ponded at control points during part supply, for
the benefit of a few new offtakes, it acquired a sand bed which
refused to scour out when full supply levels were imposed, and
prevented full supply from running. Estimate how much dis-
charge would run, with the sand bed, at 10 ft. deep, and the
minimum slope that would be required to scour the bed.

**Answer.** If the flow at 10 ft. were moving normal bed-load
then $V^2/d = 1.0$, so $V = 3.16$ ft/sec. and $Q = 3.16 \times 10 \times
150 = 5,050$ cusecs. To find the slope needed to move normal
bed-load insert this discharge, with $F_{bh} = 1.0$ in equation (7.10’’
) to obtain $S_{*} = 0.11$ approx.; if the side slopes were hydraulically
rough a little more than this slope would be needed. In any
event the canal needed about 20% more slope than it possessed,
if it was to be self-cleansing.

8.14. **A semi-regime problem.** Canals that start their history free
from bed-load may acquire one by erosion of their own boundaries
or by other means. The following illustrative problem uses regime
equations for a sectional form to which they cannot apply accu-
rately; however the answer shows roughly what to expect under
given circumstances.

**Problem 8.7.** A short power canal for 1,100 cusecs has been
designed to 20 ft. bed width, 70 ft. water surface width and
10 ft. depth. It has been excavated in a sandy material contain­
ing pebbles, and it is believed that: (i) if the bed could be
eroded the zero bed factor of the material in movement would
not exceed 1.0; (ii) the sides would stand erosive attack about
as well as a clay soil with side factor 0.3.

(a) If the water entering the canal is free from sediment, esti­
mate the chances of the canal retaining its section under
full supply conditions.

(b) If the water entering the canal carried, or acquired, a sand
load with bed factor 1.0, what would happen to the canal
section in the course of time?

**Answer.** (a) Actual $F_b = \frac{2.45^2}{10} = 0.6$, which is far enough
below 1.0 to suggest that, despite side-slopes much flatter than
in the canals behind regime equations, the bed would not move.
Actual $V^2/b = \frac{2.45^2}{45} = 0.32$, so, with steep side-slopes erosion
seems just possible, but would not extend far. However, there
would be considerable variation of attack over the actual flat
sides; a very rough criterion for side erosion near the bed
would be $\frac{2.45^2}{20} = 0.7$, suggesting that the toes of the slopes
might cut out, as observed in various designs of this type.

(b) With the given load the regime equations would apply
eventually. Bed deposition would occur first and increase $V$ and,
therefore $V^3/b$, so that sides would tend to erode. The final
adjustment would be to 6.95 feet deep and 60.5 feet broad at
half depth assuming fairly steep natural ultimate side slopes
(use equations (7.8, 9)). The regime slope would adjust but
the shortness of the channel would prevent any appreciable
changes in level. The final dimensions would fit fairly neatly
into the original channel top.

It is to be noted that, if the sides eroded in (a), they would pro­
duce material that would deposit on the bed just downstream,
enhance velocity there, and produce more erosion.
CHAPTER 9

FORMULA APPLICATIONS: FLUCTUATING CANAL FLOW WITH MODERATE BED-LOAD CHARGE OF SAND

9.1. Introduction. In the previous chapter canals of small bed-load charge were treated as practically steady at authorised full supply discharge, for in-regime purposes, because the beds became inactive as soon as flow dropped a little below that value, and deviations above full supply were very small. In this section the regime equations are applied, as a step towards understanding rivers, to canals where, as an approximation, the conditions of exact application (para 7.5) are relaxed to the extent that:

i. Discharge and bed-load charge may fluctuate, not too rapidly.

ii. Bed load is significant, so that Eq.7.5 or an extension must be used but the bed remains duned, see paras 10.4, 10.26.

This relaxation raises a couple of points that must be discussed before proceeding with applications. The applications, starting with para 9.6, are chosen to concern river operations and aim at giving a sense of proportion and some skill in computation; for simplicity Eq.7.5 will be used even when C exceeds 10.

9.2. Equilibrium equations used for non-equilibrium. In mechanics an equilibrium equation is for a virtually steady state. It may be considered as the general unsteady state equation in which the time-rate-of-change terms have been reduced to zero. As dynamic terms are continuous, an equilibrium equation can often be made to serve, to some degree of approximation, as a non-equilibrium one if the time-rate-of-change terms are small enough. (One notes, in passing, that an equilibrium equation, by itself, need give no clue to the form of the non-equilibrium one of which it is a special case). So regime equations can be used, without too much error, within some range of rate of fluctuation.

9.3. The practical question of limits to rates of fluctuation cannot be answered precisely. There is no basic theory for a non-equilibrium transport equation. Available empirical transport formulas, including the regime slope equation (7.3a), are based on steady flow transport data in small straight laboratory flumes, operating in a variety of phases, and provided, mostly, with unnatural sands and fine gravels; the interpretation of the data and the functional forms
used for the formulas differ, but the basic data pool is the same for all. Fig. 9.1, adapted from Vanoni, Brooks and Kennedy, 1961, by Blench, 1964 (a, b) shows wild divergence among alternative formulas and the scanty available field data. Routine measurement of C, in the field, is impossible at present and special observations are difficult and their interpretation is somewhat controversial. There is not even agreement on the exact definition of bed-load (para 4.20). At present, therefore, for all these reasons, limits of applicability of the regime formula for transport must be decided by patient and cautious trial applications combined with comparison of their indications with facts.

9.4. A rough guide to permissible limits of discharge fluctuation comes from laboratory flume studies. If the steady discharge in a flume producing dunes is increased suddenly by a small but significant fraction to a new steady discharge several minutes will be required for a new steady dune pattern to develop. So, if discharge were to change continuously by more than a couple of per cent per minute the dune pattern at a particular time would be expected to differ noticeably from the equilibrium pattern corresponding to the discharge at that time. Rates of change of discharge in rivers do not exceed this amount usually.

9.5. **Source of bed-load charge.** Suppose a short laboratory flume running steadily in-regime with a bed-load charge C derived by injecting it along with the fluid discharge upstream. Suppose now that the charge were stopped suddenly. Would the channel start to pick up the same charge from its bed (till degradation altered conditions)? The general engineering opinion seems to be that it would, but without definite proof that it does. In this text the general opinion is accepted with an open mind. So equation (7.3a) will be taken, provisionally, to give the charge C, for the values of the other variables (provided rates of change of variables are below some limit) whether the charge comes from upstream, or from bed erosion at the position of measurement, or both.

9.6. **Problems of assisted river cutoff.** Some problems can now be attempted. An assisted cutoff (para 2.28 and Plate 7) of a river loop is usually a straight canal, excavated much smaller than the river during a convenient stage of flow and expected, because of its excessive slope, to erode and grow larger when the river stage exceeds some amount. If the design and operational planning (Blench, 1961 (a) p. 452) are inadequate a cutoff may accrete instead of enlarging and will have to be re-excavated. One possible
step in design is to prescribe a tentative cutoff channel size and test whether its bed-load and/or bed-factor exceed those estimated roughly for the river above a selected stage, and whether the side-factor above that stage exceeds the limit of bank material resistance. If the excesses are considerable then the cutoff will enlarge and, barring new factors (para 8.9), the larger it becomes the less slope it will need and, therefore, the more unstable it will become. The following two idealized problems illustrate, inter alia, how, as the river rises without changing energy grade over a cutoff (actually there would be some change), the tendency to erode increases. The designer would not believe the C he calculates—a very good transport formula might give an answer within half and double the truth (Fig. 9.1). However he could place fair reliance on its ratio to the C of the river at the same stage as estimated by the corresponding formula properly applied; this is all he needs.

Problem 9.1. The section of a long straight channel may be approximated by a trapezoid of 100 ft. bed width and sides sloping at 2 upon 1. The bed material is sand of median size 0.275mm., and the sides may be assumed hydraulically smooth. Mean annual water temperature may be taken at 52.5°F. and calculations may be in terms of the corresponding clean water viscosity (kinematic) of $1.35 \times 10^{-5}$ ft$^2$/sec. Bed slope of channel is 0.14 per thousand. Estimate the discharge, bed-load charge, bed factor and side factor that prevail when flow is at 3.5 ft. depth, parallel to the bed, and has occurred long enough to allow the bed pattern to adjust to the conditions, but not long enough to cause erosion sufficient to alter the problem.

Answer. Notice that four quantities are to be found, so all four regime equations will be needed. We cannot start from the sectional equations (7.8, 9) since they contain three unknowns after b and d have been inserted. Equation (7.11”) is appropriate. The value of $K = 3.63g/\nu^{1/4}$ is 1,950; b is 101.75; $b^{1/4} = 3.16$; $3.5^{1/8} = 1.17$; $F_{b0} = 1.0$. So $f''(C) = 0.14 \times 1.95 \times 3.16 \times 1.17 = 1.01$, which is outside the range of Fig. 7.2; so we might estimate C as 0.1 from the approximate relation $f''(C) = 1 + 0.12C \times 7/8$ for small values of C. (If $f''(C)$ had been less than 1.0 there would have been no bed movement). Now $F_{b0} = 1.0$, so $F_{b}$, by equation (7.5), is probably about 1.012, showing that the charge is not enough to affect bed factor appreciably. The value of Q is $1.0 \times 101.75 \times 3.5^{1/2} = 665$ cusecs by equation (7.12). Flow area is $3.5 \times 101.75 = 355$ ft$^2$ so $V = 1.88$ ft/sec,
and $F_s = 1.88^3/101.75 = 0.065$, showing that this flow would not erode even loam of very poor cohesiveness (para 7.18).

**Problem 9.2.** With the data of problem (9.1), except that discharge is given as 20 times the 665 cusecs found for it and the depth has to be determined, find depth, bed-factor, side factor and bed-load charge.

**Answer.** The equation suitable is $(7.11''')$; equation $(7.11'$) is unsuitable, because we do not know $F_s$ in a problem where a flow has not run steadily for a long enough period for the sides to erode till the side factor drops to correspond to their cohesive resistance. Actually $b = 100$ inserted in equation $(7.11''')$ would suffice, but for the sake of the exercise allowance will be made for variation of $b$ with $d$. A preliminary estimate of what $d$ is likely to be can be made from the Manning formula, taking hydraulic radius $r$ as a fixed multiple of $d$. Then $V \propto d^{2/3}$ and $Q \propto d^{1/3}$ as a first approximation. So $d$ will be about $20^{2/3} \times 3.5 = 21$ ft., and $b$ about 110.5 ft., which is about 8 per cent more than in the last problem; $b^{1/6}$ will be about 1.3 per cent more. $Q^{1/12}$ will be $20^{1/12} = 1.284$ times more. Then equation $(7.11''')$, in which $S$ is constant, shows that $f'''(C)$ will be $1.284 \times 1.013 = 1.3$ times more than in problem (9.1) where, because all the $C$ functions are practically equal at small $C$, it was 1.01. Thus $f'''(C)$ is 1.31, and Fig. 7.2 shows that $C$ is 3.0. Then $F_b = 1 + 0.12 \times 3.0 = 1.36$, and the accurate value of $d$ from equation (7.12) is 22.0 ft., $V = Q/bd = 5.45$ ft/sec., and $F_s = 1.45$ which would cause any loam soil to erode.

The first problem was devised to have the bed just active, and the second to demonstrate that a very large multiplication of discharge produced a fairly small bed-load charge compared with known cases, such as ephemerol sand rivers of the Western U.S.A. where charges of the order of 100 are possible. They help to explain matters such as why periodic cessation of bed-load charge into irrigation canals has little effect on regime slopes, and why engineering calculations of scour at river bends may use the same approximate bed-factor at different high stages. A more definite appreciation of the relation of bed degradation to charge is obtained from specific cases.

**9.7. Rate of bed erosion after enhancing discharge.** The following two idealized problems suggest that, in an irrigating canal system, seasonal slope changes near the head would be very small with conditions equivalent to average charge of 0.1, but noticeable with
charge about 1.0. A cutoff canal would hardly develop rapidly, in a river with negligible bed-load charge, unless the eroded charge were of the order of 10.

**Problem 9.3.** Suppose the flow entering the reach of problem (9.1) were free from bed load, so that the C calculated must be provided by erosion of the bed within the reach. How many days would be required for the flow to pick up a volume of sediment equivalent to that in a layer 0.01 ft. thick on 1,000 ft. of bed?

**Answer.** A cusec day of water weighs 2,700 short tons, and a ton of dry sand occupies about 22 ft.\(^3\). So the daily bed load of sand moved by the flow past any section, at \( C = 0.1 \), is \( 665 \times 5.94 \times 10^4 \times 10^{-6} = 39.4 \) ft.\(^3\). The volume to be moved is 1,000 ft.\(^3\). Therefore, the time required is 25.5 days for this layer of about 3 sand-grains thick.

**Problem 9.4.** Apply the question of problem (9.3) to the data of problem (9.2).

**Answer.** The discharge is 20 times greater than in problem (9.1), and the charge is \( 3.0 / 0.1 \) times greater. The time, therefore, is \( 0.1 / (3.0 \times 20) \) times 25.5 days, or 1.02 hours. At this rate not quite 3 in. would come off the bed in 24 hours, if it were confined to the 1,000 ft. reach. If the reach were at the head of a canal, instead of being in a river, and if the discharge kept at its high level for a day, an observer would notice a little drop of head level at the end of the day, and less about 1,000 ft. downstream.

9.8. **Erosive effect of enhancing slope.** The question why a small river cutoff can capture the whole river in remarkably short time is answered by the following problems.

**Problem 9.5.** Answer problem (9.1) with the bed slope 0.42 per thousand.

**Answer.** The slope having been enhanced to 3 times that of problem (9.1), \( f''(C) = 3.03 \) and Fig. (7.2) shows \( C = 26 \). Then \( F_b = 1 + 0.12 \times 26 = 4.12 \), \( Q = (4.12)^{1/2} \times 665 = 1,360 \) cusecs, \( V = 3.85 \) ft/sec., and \( F_s = 0.56 \).

**Problem 9.6.** With the data of problem (9.5), except that discharge is given as 20 times the 1,360 cusecs found for it, and the depth has to be determined, find depth, bed factor, side factor and bed-load charge.

**Answer.** \( f'''(C) \) corresponding to \( f''(C) \) of 3.03 is 3.30 by Fig.
(7.2), and has to be increased 1.3 times, as in problem (9.2), giving \( f'''(C) = 4.30 \) and \( C = 41 \). Then \( F_b = 1 + 0.12 \times 41 = 5.92 \) and the accurate value of \( d \) from equation (7.12) is 21.7, nearly as in problem (9.2) because the proportional enhancement of \( F_b \), due to the enhancement of discharge, happens to be almost the same in both problems; on another part of the \( C \) curve agreement might not be quite so perfect. \( V = 11.3 \text{ ft/sec.} \ F_s = 14 \).

These examples illustrate that a three times increase in slope is far more effective in increasing regime charge than increasing discharge twenty times, and is more effective for a channel that has a smaller initial charge. The regime equations show the same analytically, but not so strikingly as from a numerical example. If one allows for the fact that a curved regime stream does not transport bed-load as readily as if it were straightened with the same slope, the rough rule that a chute occurs readily if the loop to chord ratio of a meander bend exceeds about 1.5 is explained.

9.9. The effectiveness of Manning’s equation. Manning’s equation remains, after some 80 years, the most popular general-purpose rough-boundary flow formula for engineers. Applied to comparison of equilibrium dimensions of canals of a system it gives \( n \) varying roughly as the inverse twelfth root of depth and is replaced in regime theory by equation (7.3b) para 7.23; however few engineers using regime theory would object to applying Manning’s equation to estimate the form of a stage-discharge relation in a canal or river. Inability to measure roughness, and many practical difficulties, make the comparison of river flow formulas generally infructuous, but 80 years of popularity seems to indicate some peculiar effectiveness, on an average. The next two problems based on the data of the preceding ones, suggest that the Manning index, 2/3, for hydraulic radius, allows for the change of bed-factor with stage so gives the Manning formula combined simplicity and effectiveness for stage-discharge relations (which, being for conditions in which fluctuation from equilibrium has occurred relatively slowly, are likely to justify practical application of equation (7.3b)).

Problem 9.7. Calculate Manning’s \( n \) for problems (9.1) and (9.2).

Answer. With \( V = 1.49 \frac{r^{2/3}}{S^{1/2}}/n \), \( n = 1.77 \times 10^{-2} \frac{r^{2/3}}{V} \). For the small discharge \( r = 355/107.8 = 3.3 \), and \( V = 1.88 \), so \( n = 0.0208 \). For the large discharge \( r = 2420/149 = 16.3 \), and \( V = 5.45 \), so \( n = 0.0208 \). This perfect agreement of the two values of \( n \) is accidental; they are usually close.
Problem 9.8. Compare the value of Manning’s n for problems (9.5) and (9.6).

Answer. They have to be in the approximate ratio of problem (9.7) because the r and V ratios hardly alter. For the small discharge n will be $3.06 \times 10^{-2} \times 2.22/3.85 = 0.0176$; for the large discharge n will be $3.06 \times 10^{-2} \times 6.4/11.3 = 0.0174$.

9.10. Analytically, Manning’s equation states $n^2$ proportional to $d^{1/3} F_b$. Substituting for d from equation (7.11”) and replacing $f''(C)$ by $f'(C)$ via the definitions of the C functions gives:

$$n \propto \frac{[f'(C)]^{1/5}}{(1+C/233)^{8/15}} \times \frac{1}{b^{1/3}} \quad \text{(9.1)}$$

which, as C generally increases with stage, shows that Manning’s n should increase with stage. (We continue to assume (para 9.1 (i) ) that regime equations will give acceptable answers to slowly changing stage). However, insertion of practical values of C for different stages show that the variation of n is slight. For narrow channels in which r would increase with stage less rapidly than d, n might even decrease slightly with stage. Tortuosity, not considered here, affects “roughness” greatly.

9.11. Practical use of mean bed-factor for varying stage. Common engineering practice is to assign a bed-factor to a river and use it for approximate calculations over a considerable range of stage. Such calculations usually concern depth—as a preliminary to estimating scour—and equation (7.12) shows that depth depends on the inverse cube root of bed-factor in a canal; so 9% error in $F_b$ would make only 3% error in an estimated depth. The following problem illustrates the small error in a large change of stage.

Problem 9.9. If an observer estimated $F_b$ of the channel of problem (9.1) from field data, with 665 cusecs running, and used this value of $F_b$ to estimate the depth with 20 times as much water running, what would be his percentage error? What would be his percentage error if he applied the same process to the heavily-laden channel of problem (9.5)?

Answer. In the former case the observer would put $F_b = 1$ in equation (7.12) instead of $F_b = 1.36$, so would be $(1.36)^{1/3} = 1.11$ times in wrong depth, i.e. 11 per cent high. In the latter he would be $(5.92/4.12)^{1/3} = 1.12$ times wrong. This error could be just enough to be annoying in canal work, but would probably be undetected in river work.
CHAPTER 10

DESIDERATA BEFORE APPLYING FORMULAS TO RIVERS

10.1. Introduction. Rivers, including models, may be treated algebraically as fluctuating canals of moderate bed-load charge like those of Chapter 9 that have been neglected long enough to meander. However, there are basic and practical difficulties such as:

i. Conditions may be far from those specified for regime theory equations in paragraph 7.5.

ii. Gravel (which include boulders) has a settlement law different from that of sand, seldom follows its simple log-normal particle size distribution (paragraph 3.7), and may include shapes never found in river sands.

iii. Mean discharges of water may be enormous in rivers or trifling in models compared with those of canals; river sediment charges may be relatively enormous. The ranges of fluctuation of water and sediment may also be relatively large.

iv. Suspended load may be both large and permanent.

v. Meandering prevents the simple geometric sections of canals.

10.2. Despite the potential difficulties, cautiously selected and verified applications of regime theory equation forms to rivers, not too far outside the limits of strict applicability and allowing for the factors of para. 10.1, have proved quantitatively useful in a wide range of river problems related to engineering and geomorphic analysis; a special reason for this success is that most rivers of major engineering interest are either sand-bed ones of relatively small bed-load charge or large gravel ones whose flood depth is much larger than bed-material size. On the other hand formulas outside regime theory, derived from observations in tiny laboratory flumes, obviously have little relevance to sand channels of the order of size of irrigation canals (see example in para. 8.4); they do concern river models and medium or small gravel rivers. Fortunately the phase (para. 4.27) picture presented by both field and laboratory (Simons & Richardson, 1966, Kellerhals, 1967, Blench, 1967 (d), Cooper & Peterson, 1968) has been coming into focus.
recently and (i) aiding amplifications of basic regime theory forms to allow for large bed-load charges, (ii) clarifying limits at which the forms change radically, (iii) suggesting forms for use beyond those limits and (iv) showing the defects of older flume experiments and analyses. Accordingly, the rest of this chapter will deal with phases, and with matters common to the application of formulas believed appropriate to them. Chapter 11 will return to mainly regime theory applications since their utility and effectiveness are well established. As with Chapter 4, which is assumed read, first perusal may be cursory.

10.3. Definitions related to bed-load transport. As terminology is not standard the following definitions supplement, or amplify, those in Chapter 4. Sediment load is the weight of sediment per unit time passing a section. Suspended load consists of particles that never rest on the bed. Bed-load is the part of total load that is not suspended. Sheet, or flat, flow is said to occur when the bed-material rolls or scurries along a flat bed in any concentration. Saltation is the sporadic jumping movement of bed particles that are continually whisked up by the flow and redeposited. So saltating material forms part of bed-load. (Some authorities prefer to exclude saltating material from bed-load and categorize it as bed-material in suspension; they would call our suspended load “wash load”.) Critical flow, as in rigid boundary hydraulics, occurs when small gravity waves on the water surface remain stationary relative to the fixed banks; supercritical flow is faster than critical and sub-critical flow is slower. Critical dimensions (e.g. depth, discharge intensity) are those associated with critical flow.

10.4. Visible phases in steady bed-load transport. As explained in paras. 4.27,28 the phases of turbulent flow of a Newtonian liquid in a uniform circular pipe are disclosed by friction-factor plots; but they are invisible. In supercritical channel flow, however, a phase where the friction-factor depends, additionally, on a Froude Number may be advertized by conspicuous surface waves. In channel flow with a mobile bed various visibly distinct wave-forms on the bed advertise different phases and warn the engineer that he may need radically different formulas to represent their causes. Special attention is drawn to these warnings since engineering literature may present equations or graphs (e.g. those of Fig. 9.1) without a statement of the bed-forms associated with their derivation or application, even if the original experimenters made one.

10.5. The classic description of bed-forms is contained in the work
Gilbert, 1914, who undertook, with remarkable effectiveness for the time, the enormous task of measuring and correlating the interrelated variables for all phases obtainable in flumes. The main description was for a sand with progressively faster steady flow conditions. The general impression from his report is that the start of bed movement is associated with the bed ruffling into unsymmetric dunes, like the sand ones of the desert. The dunes move downstream relatively slowly, maintaining their general pattern but undergoing some merging and redevelopment. Rolling and saltation are associated with them and the saltating cloud does not obviously reach the water surface. Increased velocity makes the transport vigorous but, as critical velocity is attained, the dunes flatten out and vanish. Close to critical velocity the bed is flat but, of course, covered by a cloud of scurrying particles. At some velocity above critical symmetric sinusoidal dunes form and move upstream. He named these "antidunes". The repeated piping failures of the bed due to large water-surface wave slopes at very high velocities were noted and described as paroxysmic.

A phase not mentioned by Gilbert was reported by Bhattacharya, 1960, following bridge-pier scour experiments (Varzeliotis, 1960) where a bed of 1.7 mm log-normal gravel acquired small but appreciable movement before starting to become duned. Attempts to induce dunes in this phase by making dune-like irregularities had been found to result in the flow smoothing them out. He acknowledged that this flat subcritical stage had been noticed by others. Ansley, 1963, described and filmed a "shock antidune" consisting of a surge that moved steadily upstream complete with its bed-form. A concentrated study of bed-forms in sand (Fig. 10.10), using modern equipment and techniques in a very large flume, has been made and reported with illustrated descriptions under U.S. Geological Survey auspices by Simons, Richardson, and Albertson, 1961, and Simons and Richardson, 1962, 1966 to whom Fig. 10.10 is acknowledged. This study emphasizes that there can be visibly distinct dune forms within subcritical flow. We shall call them, informally, mild and wild dunes. The authors called them "ripples" and "dunes" which may have ripples on their backs; they appear to correspond to Bagnold's (1960) two kinds of "ripples" in desert sand dunes. Plates 21, 22 illustrate. The mild dunes occur at lower velocities. The change from dunes to flat bed can occur far below critical velocity, but antidunes form close to critical velocity.

For statements about the boundaries between phases Simons
et al used modern analysis which will be clear from paras. 10.12, 13. It shows that simple statements like "flat flow changes to antiduned at x times critical velocity" are inadequate. In fact, phase changes have to be located by curves on diagrams—sometimes by curves on many sheets of diagrams—as in Fig. 10.8, adapted from Simons and Richardson, 1966. A proper statement about the division line in this Figure between ripples and dunes would be of the form "provided that d/D is unimportant, the division line between ripples and dunes is as designated on the graph of stream power against median fall diameter of particles (of a standard mix)—both imagined to be non-dimensionalized by v, g; this line terminates to the right at a particle size of 0.65 mm, indicating that ripples do not occur in coarser materials".

10.8. The net result of all investigations for purely bed-load transport phases to date is to confirm Gilbert’s phases, add a flat pre-duned one, divide the duned phase into two subphases ("mild" and "wild"), add a third subphase to antidunes, and replace Gilbert’s clear-cut phase boundaries by multi-dimensional ones that will not be well defined without improved and extended research. Modifications due to addition of suspended load, given by Simons, Richardson, and Haushild, 1963, may be striking.

10.9. Simplified phases. For most practical work only a broad knowledge of phases is necessary. So this text will refer, usually, only to flat pre-duned, duned, flat postduned and antiduned phases. Dunes will be unsymmetric bed-forms that move downstream; antidunes will be bed-forms that move upstream. The subphases will be kept in mind. A precedent for this is in rigid boundary work where, in the Moody Diagram, the turbulent transitional subphases arising from different textures of roughness are omitted (though described in texts) and the diagram is used for noncircular shapes despite subphases due to shape and is even extended, with possible large errors, to supercritical channel flow. The justification for omissions is normally that information on subphases is too poor to be used, or good enough to show that they are unimportant for the work in hand.

10.10. Phases in rivers. The use of sonic sounders on rivers and canals has supplemented laboratory information with observations that are hard to interpret because of unsteadiness, suspensions and meandering. (Unsteadiness in bed movements takes much longer to develop and die out than disturbances in a water surface. A crude
way to obtain a sense of proportion is to consider that die-out time in the two cases might be roughly in the ratio of surface wave to bed wave speed). There are reports of ripples on the backs of dunes, of macro-dunes, of some antidunes forms that move downstream in supercritical flow, of others that occur in markedly subcritical flow, and of gravel dunes that grow longer under conditions that would make sand ones grow shorter. Sources of information of various kinds include Pretious and Blench, 1950; Carey & Keller, 1957; Bagnold, 1960; Kennedy, 1963; Yalin, 1964; ASCE, 1966; Galay, 1967; Neill, 1969.

10.11. **Phase Formulas.** The picture of formulas or graphs to extend or replace regime-theory ones outside of their original range of observation has improved enough recently to merit description, though it is still imperfect. For this purpose a simple “prototype case” is presented below for bed-load conditions only. A dynamically complete verbal statement of the case is made, put into symbols and non-dimensionalized to reduce it to equations among the theoretical minimum number of variables. By using standard non-dimensional variables (numerics) implicit in regime-theory formulas the way is opened for comparing, with maximum simplicity, data plots and consequent formulas for all phases. The reader unacquainted with dimensional analysis (Yalin, 1965; Blench, 1968) can accept the general dimensionless relations without loss to the rest of his reading since their only purposes are to ensure that no essential physical factors are omitted and that the labour of plotting or formulating is a minimum.

10.12. **The prototype case.** The selected prototype case concerns a basic bed-load experimental set-up in a laboratory flume, and the consequences when it is caused to run. The statement is:

“A steady volumetric discharge $Q$ of Newtonian fluid with kinematic viscosity $\nu$ and mass density $\rho$ is diverted into a long horizontal rectangular flume of breadth $b$, whose sides behave as if hydraulically smooth. A granular material of fixed constitution is added to this discharge at a fixed rate, or “charge”, $C$, units of weight discharge per unit of fluid weight discharge. The granular material is selected with a constitution such that, when it has formed a bed, it will all move in the rolling and/or saltating manner recognized as bed-movement. Its median size is $D$ and the mass density of every particle is $\rho_s$; shapes, and the distribution of sizes and shapes, are susceptible to definition by a
host of non-dimensional factors represented by X . . . . The
motion takes place in the earth’s field of body-force, g.
When, and only when, all the above factors have been imposed
the channel will form a granular bed and adjust itself to have a
unique space-mean depth d, and a unique energy degradation
rate of gS per unit mass per unit length along the flow (cor­
responding to a surface slope S). There will be, also, a definite
mean flow velocity V and, under many circumstances, bed waves
of mean wave-length and amplitude λ, a.”

Note that the same problem could have been stated from different
equivalent viewpoints that would interchange dependent and in­
dependent variables; the present viewpoint is for sediment injection
but might have been changed to suit circulation.

10.13. The stated consequences of the preceding experiment are for
five important independent physical relations, covered by equations
of the general type:

\[ d, gS, V, \lambda, \alpha = \text{fn}(Q, v, \rho, b, C, D, \rho_s, X, \ldots, g) \]  
\[(10.1a, b, c, d, e)\]

Conventionally we shall define b,d so that \[ Q = Vbd. \] This allows
Q to be replaced by V and Eq. 10.1c to be rejected. Then con­
ventional dimensionless reduction can yield the four independent
relations, basic to the rest of this text:

\[ \frac{V^2}{gD}, \frac{V^2}{gdS}, \frac{\lambda}{D}, \frac{\alpha}{D} = \text{fn} \left( V \frac{\sqrt{\nu g D}}{\nu}, C, \frac{d}{D}, \frac{b}{d}, \right) \]  
\[(10.2a, b, d, e)\]

The set of numerics (dimensionless quantities) arose in regime
theory analysis of the classic Gilbert data by Blench and Erb, 1957.
The velocity \( \sqrt{\nu g} \) is called Vig. (for viscosity-gravity) and the
Reynolds Number formed from it, in terms of particle size, is the
particle Vig Number. s is \( \rho_s/\rho \) which, in river practice, is the specific
gravity. The regime slope equation (7.32) calls for:

\[ \frac{V^2}{gdS} = \text{fn}(Vb/\nu, C, d/D, b/d, s, X, \ldots) \]  
\[(10.2'b)\]

which can be deduced algebraically from the preceding equations,
or by restatement of the case. Further algebra (Blench, 1968) can
produce the common, alternative system where \( V_s^2 \) is used for gdS,
\( p/(\rho g V_s D) \) replaces C and p is load per unit channel breadth, and
\( V_s D/\nu \) replaces the Vig Number (Yalin, 1965). Settlement velocity
can be introduced via:

\[ V_s = \text{fn}(D, \rho, \rho_s, \nu, g, X', \ldots) \]  
\[(10.3)\]
Of course, no amount of algebraic reshuffling can alter the fact that there must be a certain number of independent equations and a certain number of interlinked variables (in general) in each. All that alters is the appearance of statements to this effect.

10.14. The derivation of formulas. Clearly, the production of plots and thence formulas from experimental data is, in general, a seven-dimensional task (counting $X\ldots$, which represents something like "texture" in rigid boundary problems, as one variable for convenience) even under the ideal conditions of the prototype case. So there is little hope of finding reliable regime or transport formulas from the data unless the conditions of experiment or observation make only one or two independent numerics important at a time; then those found for couples of variables may not link readily into a seven-dimensional expression. The following paragraphs will outline the conditions for the regime theory observations of Lacey, the special gravel river observations of Kellerhals, and laboratory flume experiments; they will discuss the formulas obtained and the phases disclosed. For brevity and because measurements of wave dimensions are scanty, attention will be focussed on the two degrees of freedom associated with $F_0,S$. For clarity $F_{bo}$ will receive attention before $F_b$.

10.15. Phase of basic regime theory. The channels subjected to Lacey's basic analysis were very favourable in terms of Eqs. 10.2a, b. Judged by the markedly duned beds there was only one phase. They were selected for having settled to long-term equilibrium after many years. Analysis was at "full supply" and, as the data were collected at fairly random times, plotting was likely to average out fluctuations about long-term equilibrium, including those due to seasonal suspended load. The small $C$ (inferred not measured), and the method of regulation, made authorized full supply discharge a good assessment of the equivalent uniform formative one, and the smallness of $C$ also left its effect undetected in his analysis and in a later one by Bose, 1939, who used $D$ instead of Lacey's $f$. The bed-material being sand, of "universal" grain-size distribution, would have practically the same $s,X\ldots$ for all systems. Within a system $b/d$ varies as the sixth root of $Q$, that is by a little more than 2 times (increased by scatter due to variations of $F_b$ and $F_s$) for a discharge range of 100 times, and $b/d = 6$ would apply to a tiny channel like a large field-ditch. Channels were analyzed initially by systems (each with its own
takeoff from a river) to smooth out the effects of differentiation of sediment among canals of each system.

10.16. Under these circumstances Eqs. 10.2a, b say that smoothing curve formulas like:

\[ F_{b0}/g = \frac{V^2}{gd}, \frac{V^2}{gds} = \text{fns (} \sqrt{\frac{V}{gD}}, \frac{d}{D} \text{)} \]  

(10.2a", b")

should have been expected, and would correspond to equivalent uniform flow conditions. (In all dimensionless equations “d” will denote “a measure of depth”, for example hydraulic radius, depth from water to smoothed bed, etc., and bd will measure cross-sectional area of flow; this covers Lacey’s initial use of hydraulic radius and wetted perimeter). In fact (para. 7.3i) d/D turned out to be irrelevant in Eq. 10.2a". So the bed-factor relation was two-dimensional and the original Lacey friction factor one three-dimensional in terms of numerics. The generalized formulas of this text leave the number of numerics unchanged, if, for physical reasons (Blench, 1969), the two-dimensional King Equation 7.3 which follows from Eq. 10.2b’ is written as:

\[ \frac{V^2}{gds} = \text{fn (} \frac{Vb}{\nu}, \frac{b}{d} \text{)} \]  

(10.4)

with the understanding that then the effect of b/d was not noticeable in the small range of b/d > 5 that occurs in any one canal system. (It is noticeable with the larger range of b/d > 2 that occurs in laboratory flume experiments.) The side-factor equation is not covered by this discussion as b is not self-adjusting for the rigid sides of the prototype case.

10.17. **Graph for \( F_{b0} \).** Fig. 10.4 shows \( F_{b0} \) against d/D. The two firm horizontal lines at the bottom right show the approximate limits of verification of regime theory; the dotted extensions indicate successful application to rivers, see Fig. 3.3. The line marked “probable 5” gravel” is from Eq. 10.12 so is rather speculative. Though a rather small part of the diagram is covered, the practical field coverage concerns (i) all sand channels from a few cusecs up to several million and (ii) most large gravel rivers. The effect of appreciable C can be built on the foundation laid by \( F_{b0} \).

10.18. **Subcritical unduned phase of Kellerhals.** Kellerhals, 1967, made a regime theory type of analysis for the three degrees of freedom of selected gravel rivers draining lakes and believed to have acquired equilibrium with \( C \to 0 \) at about high flood discharge. Dunes were absent. The gravel, having had its fines
leached out long ago, had approximately log-normal particle size distribution but with dispersion (probably associated with the particle shapes) considerably greater than for sand. Gravel sizes ranged from about an inch to one foot, so these observations are the first to break away from the tiny range of small particles common to flume experiments (see para. 10.22) and Lacey canals. He supplemented his data mainly from supposedly comparable gravel canals. The b/d values scattered randomly in the general range 20-30, so b/d should be even less important than in the canals of regime theory. Expectation purely from Eqs. 10.2a, b is therefore Eqs. 10.2a", b", with X . . . added to the independent variables.

10.19. Because bed-load charge had vanished and the beds were flat the friction-factor formula should be expected to be as for rigid bed, with D (qualified by X . . .) measuring roughness height. This is what he found with the scatter of points sufficient to allow a choice among the common competing flow formulas—Manning, Manning with the index of d replaced by 3/4, and logarithmic. The regime theory formula has index 3/4 but its equivalent roughness height decreases as D increases (see Eq. 7.3c), in agreement with the fact that roughness does in fact decrease to suit the flattening of dunes.

10.20. When his slope data are plotted against d/D and used with:

\[ \frac{V^2}{gdS} \alpha \left( \frac{d}{D} \right)^{1/2} \]  

(10.5)

the result is:

\[ \tau \alpha gdS \alpha D \]  

(10.6)

which is the finding from the flume experiments of Meyer-Peter, 1948, and Shields, 1936, both of whom made a reasonable adjustment of their data to correct for the effect of b/d. His own direct plots of shear stress scatter too much to decide definitely on the applicability of Eq. 10.6. Combining Eqs. 10.5,6:

\[ F_{bo}/g = \frac{V^2}{gd} \alpha \left( D/d \right)^{1/2} \]  

(10.7)

The band on Fig. 10.4, suitting this equation and marked “Kellerhals and Lane”, contains his semi-smoothed points. Their positions had no obvious dependence on particle size; however, the possibility of dependence should not be excluded since (Eq. 10.12) the relation might be a fourth root one which could be masked by scatter. Much of the large scatter (worse in the raw data) is probably due to incorrect assessment of the discharges at which the beds would be active.
10.21. The most accurate finding was for breadth adjustment (with which the prototype case in not concerned) viz:

\[ b = 1.8Q^{1/2} \] \hspace{1cm} (10.8)

The original Lacey equation recommended for sand-bed rivers with cohesive sides was:

\[ b = 2.68Q^{1/2} \] \hspace{1cm} (10.9)

The Leopold and Maddock, 1963, (Para, 11.11) results averaged to the same functional form, with different coefficients for different river systems. The form is that of Eq. 7.8 with a specific \( F_{b}/F_{s} \). The reasons for its applicability, apparently regardless of phase, are not understood yet.

10.22. **Phase of subcritical laboratory flume experiments with \( C \to 0 \).** Hoque, 1968, plotted \( F_{b/o} \) for \( C \) less than 2 (parts per 100,000 by weight) and b/d greater than 5, from a collection of world data at all charges by Peterson and Cooper, 1968. All plots were against d/D and points were marked by values of C, then D and b/d; special plots were added for b/d down to 2.0 and C up to 10. Many particle size distributions differed significantly from that for log-normal river-bed sand; some differences were extreme. About half the median sizes exceeded 0.9 mm; the smallest median was 0.15 mm and the largest 15 mm, but 85\% were between 0.3 mm and 4 mm; only 2\% were smaller than 0.2 mm. Information on bed-forms was unsatisfactory. The range of b/d was from 1.1 to 63. Under these conditions:

\[ F_{b/o}/g = fn \left( \sqrt{\frac{g}{v}}, \frac{d}{D}, \frac{b}{d}, X \ldots \right) \] \hspace{1cm} (10.10)

for quartzose materials, and s would be significant for auxiliary data where non-quartzose materials were used.

10.23. **Multiple regression.** A digression is needed here to clarify a common cause of error in multidimensional analysis. Experiments or observations related to Eqs. 10.2 have usually compelled, or accidentally resulted in, relations among the independent variables. If this happens analysis can produce nothing more than a locus in the multi-dimensional representation of the general answer; normally the locus will be different for different experiments or observations. The point can be understood from the case of a surveyor of a mountain area whose task is to obtain values of \( z(F_{b/o}) \) at many points \( x(d/D) \), \( y(b/d) \) so that a map can be prepared. However, he is under orders to keep strictly to the line \( x = ay \). This restriction is obviously that of a flume experimenter with one flume and
one material size. Then \( z \) on the line \( x = ay \) can be expressed as a \( fn \) of \( x \) alone, or \( y \) alone, but has no definite meaning as a function of both \( x \) and \( y \); there is no way of relating the scatter arising in resurveys of \( z \) to either \( x \) or \( y \); and there is no way to deduce what another surveyor, following another line, say \( x = 3y \), would find as a \( z, x \) or \( z, y \) relation. If there are enough survey lines to cover the area well then the form of the surface can be deduced. The application to the lines of Fig. 10.5 is that their slopes should not be accepted unless \( b/d, d/D \) and \( \sqrt{\gamma vD/N} \) have been shown to be unassociated; this check was not performed as the investigation was limited. In the regime-theory and Kellerhals data there is no marked association among the independent variables. In the Peterson and Cooper data, para. 10.26, association between \( d/D \) and \( \sqrt{\gamma vD/N} \) was graphed to permit later interpretation.

10.24. **Extended formula for subcritical \( F_{\text{bo}} \).** Fig. 10.5 shows a sample, arising from the Hoque study, of bands enclosing data of selected experimenters. Some "initiation of motion" data from laboratory gravel are added to test whether they show the same trend; they do. A band curving into the horizontal was used for very large flume data that should have been in, or close to, the regime theory phase; all other data were fitted by bands of slope close to minus 0.8. The individual bands would suffer considerable change of slope if corrected for association between \( b/d \) and \( d/D \) (para. 10.23). A reanalysis (Blench 1968b) of all relevant grouped source data (Peterson and Cooper) showed a little association between \( b/d \) and \( d/D \), so experimenters generally must have covered a fair range of \( b/D \). However a double log plot of group median values of \( d/D \) against \( D \) gave a curved line of average slope about 60%; this, by para. 10.23, makes the 0.8 band slopes meaningless. The hypothesis:

\[
F_{\text{bo}} \propto (D/d)^{1/2} \cdot fn(D) \tag{10.11}
\]

tested by plotting values of \( fn(D) \) against \( D \) showed that \( fn(D) \) followed the \( F_{\text{bo}} \) curve of Fig. 7.3 reasonably. It is possible, therefore, that an experiment (if practicable) with (i) large \( b/d \), (ii) one \( D \) not larger than a few millimeters of one standard log-normal material, and (iii) \( d/D \) ranging from about 5 to infinity, would plot in Fig. 10.4 as a minus 0.5 line that flattened in the general range \( 100 < d/D < 1,000 \) to become horizontal in the regime theory phase (that probably extends to infinity). With another size of
the same material the whole line might displace vertically by the amount appropriate to the regime theory (horizontal) $F_{bo}$, as defined by Fig. 7.3; but see para. 10.49. Changes in $X \ldots$, if extreme enough, could affect positions inclinations and transitions. The possibility seems good enough to justify the practical use of lines, such as described, with tentative transitions, for $D$ less than a few millimeters, to determine $F_{bo}$ for all values of $d/D$ in subcritical flow. It also emphasizes the need for experiments free from association among the independent variables, with standard bed-material that cannot change constitution with stage, and with coverage of the whole practical ranges of $d/D$ and $D$; bed-forms should be reported in such experiments.

10.25. Phases connected with C. Figs. 10.6, 7 are samples from sheets of an analysis of world flume data by Peterson and Cooper, 1968, using numerics suggested by Eqs. 10.2a, b. The two selected dependent variables were effectively $F_{l}$, and $S$—the latter because $V^2/gdS$ is needlessly complex. $C$ was chosen as abscissa and separate graphs were allocated to discrete values (means of small ranges) of $d/D$. The graphs were arranged in sets in which points were marked distinctively for different (i) $\sqrt{V_{eg}D/v}$ (ii) $b/d$ (iii) gradings (iv) phases. $d/D$ was plotted against $D$ to check for association. Detailed study of the results emphasizes the need for improved flume experiments, and the addition of field ones to cover river gravel sizes. Generally all observed detailed phases of Simons and Richardson, 1966, except preduned flat-bed are reflected in the $V^2/gd$ against $C$ graphs for sands, which occur mainly for $d/D$ greater than 100; antidunes start close to critical velocity. The graphs for gravels ($D$ greater than about 0.9 mm), which occur mainly for $d/D$ less than 100, show an obvious change only between preduned flat bed and dunes till $d/D$ becomes very small, when a marked steepening occurs for antidunes (with sands the graphs flatten for antidunes). The $S$ against $C$ curves show an obvious change only between predunes (flat or ripples) and dunes. Changes, generally, can be formalized by discontinuities in slope, but they could all be smoothed.

10.26. Extended application of regime theory slope formulas. Plot (e) of Fig. 10.7 is fitted well by Eq. 7.11, with $F_{bo} = 0.85$ corresponding to Eq. 7.6, up to $C = 20$. All the Peterson and Cooper $S$ against $C$ plots from $d/D = 100$ to 1,000 have almost identical shapes. Finally, Eq. 7.11 fits them reasonably up to $C = 20$ provided
$F_{bo}$ is the value indicated by their $V^2/gd$ against $C$ plots at small $C$ instead of by Fig. 7.3, so is the value given approximately by the extended Eq. 10.11. This utility of a regime theory formula for conditions where $F_{bo}$ is obtained from an apparently non-regime-theory phase can be explained by the bed-forms in that phase still being duned in the general sense of para. 10.9. Eqs. 7.10,11 are nothing but the basic Eq. 7.3, which is of Blasius "self-formed boundary" type, with $V^2/d$ replaced by the rough formula found adequate for original regime-theory conditions which were for duned bed and small $C$. There is reason (para. 14.3) to associate the Blasius kind of formula with periodicity in the bed-layer even when the boundary is rigid and there is no sediment. So there is nothing surprising in an extended formula for $F_{bo}$ at any $C$ satisfying the slope equations as long as the bed has periodicity, nor in indications from the Cooper and Peterson plots that extension to large $D$ values of the experiments, where the bed appears to become flat, is not permissible.

10.27. **Amendment to the C functions of Fig. 7.2.** If we accept, provisionally, the possibility that the regime-theory slope equations can be used with $F_{bo}$ extended to suit para. 10.24, provided the bed is duned, then the C functions of Fig. 7.2 deserve modification to suit the Cooper and Peterson plots. The dotted line represents the latter and cannot be split into three because the plots could not allow for $b/d$. It is unlikely to be perfect, but the three Blench-Erb, 1957, lines for $C \to 10$ had relatively little data and were admitted to rest on imperfect derivation. For practical calculations, therefore, the latter lines should be biased well towards the former pending experiments of the type outlined at the end of para. 10.24.

10.28. Preceding the four-dimensional analysis of world data, in terms of basic numerics, just described, Maddock, 1966, performed two-dimensional analysis of data of Gilbert (1914), USWES (1935) and Colorado State University (1966) in terms of complex numerics arrived at by various arguments. The plots had to be separated into low, medium, and high velocity ones, thereby demonstrating the need for different formulations for different visible phases. The basic numerics were different from those of Eqs. 10.2, and included $V_s^2/(s-l)gD$, which has a major advantage of being fairly constant for a gravel of given $X \ldots$ in a given fluid. The present text retains $v$ instead of $V_s$ mainly because $V_s$ is not measured for gravels and most researchers are used to thinking in terms of a Reynolds Number.
10.29. Blench and Erb, 1957, without computer aid, analysed the whole Gilbert, 1914, data to test for the expected discontinuity near critical velocity that non-regime-theory formulas ignored. Although they made the exploratory dimensional analysis giving Eqs. 10.2, and, therefore, admitted the importance of \( \frac{d}{D} \) in some phase, they proceeded to analyse \( \frac{(F_b - F_0)}{F_{bu}} \) in terms of \( C \) with the hope that \( \frac{d}{D} \) might be hidden in \( F_{bu} \); and, as the data were not suitable for finding \( F_{bu} \) by extrapolation, they used \( F_{bu} \) from Eq. 7.6. The discontinuity of dimensional plots near critical velocity was so marked that dimensionless co-ordinates had to be changed to suit supercritical flow. Figs 10.1,2 illustrate. The results were adjusted in terms of small-charge experiments with natural sands to give Eq. 7.5, which does not pretend to accuracy beyond \( C \approx 10 \). Fig. 10.3 shows the preceding discontinuity tested via Eq. 7.3.

10.30. Simons and Richardson, 1966, made a major contribution by carefully controlled large-scale flume tests, of small canal size, in which detailed observations of sand-bed forms paralleled measurements. From this information, and some field data of canals and rivers with sand beds, they were able to prepare a tentative phase location chart (para. 10.6) of which Fig. 10.8 is a minor adaptation; an interesting disclosure is that, above a certain \( D \), ripples (mild dunes) do not occur. Figs. 10.6,7 present their flume information differently. Fig. 10.9, prepared by them from their data, is for estimating the ratio of channel slope to that of a congruent channel with an unmoving flat bed of the same material. Co-ordinates in the Figures are really dimensionless, but \( v, g \) have been omitted for practical simplification (just as \( g \) is omitted from the Froude Number in regime theory to give a simple bed-factor). \( C \) does not appear in the co-ordinates, although, of course, it is implicit in them; its explicit presence would prevent the use of field data where \( C \) is unmeasureable.

PRACTICAL DEVIATIONS FROM PROTOTYPE CASE

10.31. Reality compared with the prototype case. The preceding prototype phases provide a framework against which real occurrences may be studied. The major factors causing deviations from the ideal in rivers are interlinked, but the next few paragraphs will list important ones separately.

10.32. Effects of X . . . factors in gravel rivers. The average gravel river displays visibly different material sizes spread over its bed,
and a sample taken at one point normally shows a grain size distribution about the mean quite different from the log-normal kind for sand rivers (Figs. 3.4 and para. 3.7). Moreover, very different particle shapes may be mixed in one river; for example, glacial gravel from the upper catchment may be mixed with pillow-shaped limestone blocks originating from one stratum of a canyon and dimpled pear-shaped stones from another stratum. A wide range of sizes usually results in the smaller ones going into suspension at moderate discharges and the medium ones constituting bed-load (para. 10.3); at larger flows larger material goes into suspension, and larger material also take part in bed-movement; finally the coarsest materials of the bed are all moving. The prototype case was worded carefully to permit only X . . . factors that would not imply suspension since effects may be profound and various; but, even with formal suspension precluded some bed-load mixes can behave curiously. Kellerhals, 1967, described an aggrading experimental bed where sand moved along with stones in the neighbourhood of the bed, instead of skipping into suspension, and where stones rolled over sand deposit. Bagnold, 1954, deals basically with the behaviour of regular bodies in bed-movement and shows how the mix of water and solids behaves as a non-Newtonian fluid. So there is no doubt that, even without suspension, different mixes of material of different shapes, having the same mean sample size and charge, may cause different resistance to a given discharge.

10.33. **Effect of suspensions.** Suspended load may come from tributaries, or from the stirring up of bed material during high supplies. The effects may be major. Simple examples are in para. 5.21. Simons, Richardson and Haushild, 1963, found that colloidal material introduced into the flow in a sand-bed flume infiltrated the bed-material, reduced dunes sizes and changed dune shapes so as to affect roughness markedly; also, such suspensions change the apparent viscosity of the fluid and, therefore, the settlement diameter of the bed-material. King, 1947, calculated the monthly means values of $\nu$, over several years, from a set of irrigation canals, by using his Eq. 7.3. The viscosity followed the usual temperature relation for clean water till the flood season when the canals received up to 1% suspension of clay, silt and fine sand sizes; then it dropped by some 30%. A mining engineer’s trick for increasing bed-load capacity in a flume is to charge the wash-water with clay slurry; this can more than double transport. Chemical engineers study suspensions of higher concentrations than those that
concern the river engineer and have a large literature on effect; Ansley, 1963, and Subramanya, 1965, give references. Non-colloidal suspensions should not behave in the same way as colloidal as their particles have no inter-molecular attractions. Short-term regime analysis of Lacey-type canals show suspension effects clearly (Ahmed and Rahman, 1962).

10.34. Effects of meandering and braiding. For quantitative purposes of this text a meandering or braided river can be regarded as the consequence of neglecting a canal of varying discharge and load, or building it so wide that it deposits alternate shoals. The main practical results are a lengthening and a steepening due to curvature of flow in plan. Because meandering is systematic, Chapter 2, one should expect the river slope (along the deep water channel) to be a multiple, depending on meander or braid form, of the slope of the imagined original straight canal. The details associated with meandering are too vast for simple treatment; Leopold and Wolman, 1967a and Friedkin, 1945 give major field and model information.

10.35. Use of functional forms of steady flow equations. The practice of inserting equilibrium or formative values (para. 4.14) of river variables into a steady flow canal equation, and assigning a coefficient to the equation (as demonstrated above for river slope), is accepted intuitively by many engineers. To justify and learn how to control it consider a meandering river that has a couple of canal-like reaches (some rivers possess none but could be imagined to have been given them artificially). Such a canal can be treated according to Chapter 9, and will have bed-factors, side-factors and charges at formative discharge and at off-equilibrium discharges. At formative conditions the “canal” charge ought to be long-term value for the river as a whole; off formative discharge the meaning is not so definite. The average slope along the deep-water channel, of several meander bends adjacent to a canalized reach, would be some multiple of the canal slope, and the multiple would obviously depend principally on meander pattern. The time-mean bed-factor for the exaggerated sectional shape in a bed has no clear meaning since bed-factor was defined for trapezoidal shapes, but the time-mean bed-factor in the canal would be legitimate and useable to deduce canal slope (if slope could not be measured); $V^2$ divided by mean $d$ in a bend would probably differ from canal bed-factor at the same stage by a coefficient depending on skewness. The side-factor of the “canal” at a stage where bank-erosion started would
give some idea of the erodibility of banks in bends. So, on the whole we can regard the use of steady flow forms with coefficients as nothing more alarming than setting measureable real occurrences, where possible, into correspondence with the ones to expect from an in-regime canal having plausibly averaged parameters of sediment-complex, side erodibility and discharge that define the overall river behaviour. An experienced engineering observer is not likely to go far wrong in this process.

10.36. **The meander slope-correction coefficient, k.** The great practical utility of the regime slope equations 7.10,11, explained in para. 8.6, still holds for rivers, with the right hand sides multiplied by a coefficient, k, and the proper equilibrium values of the variables inserted. With these values the author believes that \( k = 1.25, 2.0, 3.0 \) and \( 4.0 \) are respectively of the right order of magnitude for a river that looks straight in an air-photo, for conspicuous well-developed meandering without braiding, for braiding, and for extreme braiding; the values allow for extra factors like ordinary amounts of snags in the bed, roots and irregularities in the banks. The k values would apply also to the proper slope equations for phases not fitted by Eqs. 7.10, 11. For braiding Eqs. 7.10′,11′ are better than 7.10″, 11″ since b has little meaning. In gravel rivers \( F_{b_0} \) is the hard term to guess and, of course, guesses at the C functions are impossible; the formulas will be used, normally, to assess whether C functions are important and then to improve estimates of \( F_b \). Whatever slope formula applies outside the regime theory phase the k values above should apply.

10.37. **Definitions of D in gravel rivers.** The main use of D is in establishing \( F_{b_0} \), or a quantity containing \( F_{b_0} \) explicity or implicity. In sand rivers D, at a point, is generally the “median diameter by weight” of a sample, meaning the size of particle that is exceeded in half the weight of the sample (neglecting itself) and, therefore, is defected in the other half. The definition arises from the practice of either weighing material between sieves, or cumulating volumes in a settlement tube. In coarse gravel rivers the stones in a sample are usually counted and, unfortunately, not weighed, so one common measure of D is the “median by number”, which is the layman’s “the median” determined by laying the stones in ascending size order and taking the middle one (or the mean of the middle two if there is an even number). Obviously the big stones on one side of the median by number weight much more than the equal number of small ones on the other side, so the median size by
weight is larger than the median by number; for an average fairly log-normal coarse gravel the former is of the order of twice the latter. To complicate matters further some writers feel, reasonably, that the median by number usually looks rather insignificant in terms of the sample’s part in resisting flow, so use a bigger size such as the 90%-smaller-than, or 98%-smaller-than, (still by number) as D for insertion into formulas. If gravels normally followed a simple standard dispersion law like sands the differences would be academic, since any particular %-smaller size would be a definite multiple of any other; but for miscellaneous gravel samples deduction of one kind of mean from another is impossible without the original grain-size analysis.

10.38. A “median diameter by area” arises from an ingenious and attractive method of sampling by Wolman, 1954. Here stones of a coarse gravel are selected at random from an area of river bed by laying down a rectangular grid of tapes and selecting the stones that fall under the tape intersections; for sampling under water that is not too deep the grid can be replaced by wading barefooted and picking up the stones that fall under the big toe at every step. The tape method ought to result in picking stones with frequencies proportional to area occupied, so an analysis by counting gives frequency “weighted by area”.

**Problem 10.1.** Suppose equal numbers of stones in a large sample center round 1, 2, 3, 4, 5 feet diameter. Estimate approximately the sample median diameters by number, area and weight.

**Answer.** An analyst using “by weight”, who counted a 1 foot stone as “one”, would “count” a D foot stone as D³ for frequency analysis. Also, the median stone is item \((n + 1) / 2\) in order of size in an odd number \(n\) (real or fictitous). So the answer lies in the following table:

<table>
<thead>
<tr>
<th>Size</th>
<th>(N^o)</th>
<th>(\Sigma)</th>
<th>Area “(N^o)”</th>
<th>(\Sigma)</th>
<th>Weight “(N^o)”</th>
<th>(\Sigma)</th>
</tr>
</thead>
<tbody>
<tr>
<td>D</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>2</td>
<td>4</td>
<td>5</td>
<td>8</td>
<td>9</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>3</td>
<td>9</td>
<td>14</td>
<td>27</td>
<td>36</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>4</td>
<td>16</td>
<td>30</td>
<td>64</td>
<td>100</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>5</td>
<td>25</td>
<td>55</td>
<td>125</td>
<td>225</td>
</tr>
<tr>
<td>Med</td>
<td>—</td>
<td>3</td>
<td>—</td>
<td>4—</td>
<td>—</td>
<td>4+</td>
</tr>
</tbody>
</table>
That is, the median stone size by area would correspond to item 
\((55 + 1)/2 = 28\) by “area number” so would be a little less than 4 feet. The median by weight would be the \(226/2 = 113\)th, so a little larger than 4 feet.

10.39. Gravels from deep channel during high flood appear to have fairly logarithmic normal particle size distribution (to any system of numbering) in many cases, and so do gravels from self-formed channels that exit from lakes and have lost their loads during geological time. The dispersion may be appreciably more than in Fig. 3.4 (for sands) and is probably dependent on particle shape. Such a gravel might have its 98% finer and 90% finer sizes by area respectively 3 and 2 times the median size by area, and the median by weight about 1.25 times the median by area and 2.2 times the median by number. A fairly log-normal gravel is shown in Plate 15b and is very different from that in the cut of Plate 15a through material that must have been shoal at some time.

10.40. Inability to measure X . . . factors leads to individual preferences for %-finer values in formulas. For example, if a formula is based on observations with a natural log-normal material and uses median D by weight an experimenter who uses a material that has a relatively large proportion of larger stones will find that the formula fits facts better if he uses, say, the 65%-finer size for D in the formula. This text uses median by weight, unless otherwise stated, for the sake of standardization.

10.41. Surface and grab samples of gravel. The deep-water channel of a gravel river during ordinary flows usually exhibits “paving”, in the sense that fines have been washed out and stones may even have been displaced into an imbricated pattern. Beneath this veneer there will be more fines. So even if D is found by weight from a sample of all the stones in a given surface area, the value will tend to be larger than from a sample taken by grabbing a volume from some depth below the surface. Further, in very high flood the pavement is likely to be mobilized and is then not representative of the active bed. Such a flood may also expose and move very large material that is normally buried; so even the ordinary flow grab sample is not representative.

10.42. Solution of problems by high-stage \(F_{100}\). If the fines in a widely dispersed gravel bed go into suspension and leave only the coarsest material on the bed at high flood, and if suspension of
gravels has the secondary effect on regime formulas that ordinary sand-silt-clay suspensions do, then analysis at high flood seems justified in terms of the coarse part of deep bed-material; the bedload charge of this material is likely to be very small. Experience has shown that such analysis often explains river behavior reasonably well for engineering purposes, despite its crudity. Moreover, most engineering problems concern long-term regime or else high flood conditions. Therefore, it is recommended that, if gravel sizes at low flow are widely dispersed so that a considerable portion of the material goes into suspension at higher flows, then analysis should be done at high flows where the bed-material can be assumed to consist of the coarse portion of the thalweg gravel sizes that follow an approximately log-normal distribution, and to have small charge. However, some gravel rivers present fairly log-normal material at all stages and have large C at the higher stages.

10.43. **Samples of gravel for finding F_{10}**. If the top size portion of a gravel has to be used for assessing F_{10} at high stage, as recommended, some trouble may have to be taken to obtain samples. In very wild gravel streams the gravel sizes that move at high stage may be exposed only at wide intervals during low flow and be buried elsewhere. As sampling during flood may be hazardous excavation will be necessary at suitable places, including the thalweg during low flow and recently abandoned channels; the coarse material discovered should be compared with the coarsest found exposed in isolated parts of the stream bed. Inquiries about its movement should be made from people who know the river. The possibility that the coarse material may not belong to the present active river should be considered. Accepted samples containing the coarse gravel should be frequency analyzed and, if they are well off log-normal, fines should be eliminated from the analysis till the residue follows log-normal distribution fairly well; however, the size range should not be cut to less than about 25 times for large gravel or 9 times for very fine gravel. The median by weight of the residue should be accepted to find D. Although the method seems crude the extreme insensitivity of F_{10} to D in equation 10.12 para. 10.44, makes it practically useful. Obviously, enough samples should be obtained to establish a stable mean D for a reach.

10.44. **Rough formula for zero bed-factor, F_{10}, of gravel at large d/D.** Following Blench and Qureshi, 1964, a crude tentative formula is recommended for practical use at large enough d/D:

96
\[ F_{bo} = 7.3 w D_r^{1/4} \left( \nu_{70}/\nu \right)^{1/6} \]  

for all materials of specific gravity about 2.65 larger than 2 mm.  

\( m_w D_r \) is the median-by-weight spherical diameter (para. 7.15) in feet, of material that has been deliberately normalized (if it is not naturally so) according to para. 10.43. If the median has been obtained by number or by area, the mechanical analysis curve must be used to deduce the median by weight; the conversion factors will probably be about 2.0 and 1.25 respectively; \( \nu_{70} \) means kinematic viscosity at 70 degrees F, and the whole viscosity term may be assumed 1.0 for everything except research purposes. The formula has grown from a succession of analyses of large gravel rivers with fair hydrologic data, to find what bed-factors, inserted in regime theory formulas, would explain their behaviours. Although there is some hydraulic reason, discussed in the reference, behind its functional form, it is best regarded as a formulated rule-of-thumb arising out of field experience. For the value of \( d/D \) above which the formula applies Fig. 10.4 is available but unavoidably leaves the user to provide transitions. It suggests that, for 4" gravel the formula starts to go wrong as depth falls below 30 feet. Fig. 7.2 shows \( F_{bo} \) for both sand and gravel. The formula is far more useful than its nature suggests, for, as its states, \( F_{bo} \) is very insensitive to gravel size.

**Problem 10.2.** Observer A measures stone sizes in a gravel sample by a system that gives values 21% greater than obtained by B. By what % does his assessment of \( F_{bo} \) exceed B's? If he estimated scoured depth round an obstacle in terms of \( (q^2/F_{bo})^{1/3} \) how much would his estimate fall short of B's?

**Answer.** Since \( 1.21^{1/4} = 1.05 \), A's value of \( F_{bo} \) would exceed B's by 5%. His assessment of scour would be less by \( 5/3 = 1.67\% \).

In fact, the practical engineer can assess \( F_{bo} \) like he would Manning's \( n \), by looking at the material and remembering the \( F_{bo} \) that fitted similar material elsewhere.

**FORMULATION OF FLAT BED PHASE AT LOW CHARGE**

10.45. **Formulas and graphs outside regime theory.** The definite practical question now arise "What formulas or graphs, with reasonable logic and factual background, can be applied to gravel rivers working outside the regime theory phase?" The answer is
that, to allow for the three degrees of freedom for breadth, depth and slope, a fairly good set of three can be obtained for the implied unduned bed if flow is subcritical and charge is small enough not to enter the equations. For supercritical flow, and appreciable charge, factual background is poor. The fourth degree of freedom, for meander dimensions, can be treated roughly by Eq. 3.1 regardless of phase.

10.46. **Gravel rivers with negligible charge.** Kellerhals views on the three suitable equations (above), paras 10.18-21, are accepted for the following recommendations:

i. **The Friction-Factor, or Flow, Formula** has the same form as in rigid-boundary hydraulics, since the bed is flat. So there is a free choice among the forms \( V/V_\alpha = A + B \log d/D = C (d/D)^x \), where \( x = 1/6 \) is for the Manning equation, and \( x = 1/4 \) is for the regime theory type; with suitable constants they gave practically the same answers from \( d/D = 2 \) to 100. The first applies the Prandtl theoretical velocity distribution for infinite depth despite his warning that it is non-rigorous for finite depth; the second is empirical; the third is empirical but has some pretensions to universality (Chapter 14).

ii. **The Shear-Stress Formula**, which concerns keeping the bed-material just mobile, is of the Meyer-Peter and Shields type, \( \tau \propto \text{gd}S \propto D \), generalized to \( \text{gd}S \propto D^{1+n} \). (\( n = 0 \) seems to have been imposed by a failure to state the prototype problem completely, and the experimental range of \( D \) would hardly have detected \( n \).)

iii. **The Breadth Adjustment Formula** is that of Lacey \( b = XQ^{1/2} \), where \( X \) depends on the type of the river system. This has been verified so often, for all types of regime canals and rivers, that there is no obvious alternative.

10.47. Specifically the forms of equations used by Kellerhals for deriving further equations suitable for practical analysis and design, restated for easy reference, are:

- Friction Factor \( \frac{V}{V_\alpha} \propto (d/D)^{1/4} \) \( \text{(10.13)} \)
- Shear Stress \( \propto \text{gd}S \propto D^{1+n} \) \( \text{(10.14)} \)
- Breadth Adjustment \( b = XQ^{1/2} \) \( \text{(10.15)} \)

Some simple but tedious algebra transforms them, via \( Q = Vbd \), to:

\[
\begin{align*}
\text{d} & \propto D^{-1/5} - \frac{2n}{5} \cdot Q^{2/5} & \text{(10.16)} \\
S & \propto D^{6/5} + \frac{7n}{5} \cdot Q^{-2/5} & \text{(10.17)} \\
V & \propto D^{1/5} + \frac{2n}{5} \cdot Q^{1/10} & \text{(10.18)}
\end{align*}
\]

which yield the very useful ones:
\[ S \propto (D/d) \cdot D^n \quad \text{(10.19)} \]
\[ F_{bo} = V^2/d \propto (D/d)^{1/2} \cdot D^n \quad \text{(10.20)} \]

If \( D \) is \( D_{90} \) by area for the Kellerhals materials, for which \( D_{90}/D_{50} \) (by any system) varied between about 1.4 to 3, with a fair average at 2, and if \( n \) has his value of minus 1/5, his coefficients in Eqs. 10.16-19 are, in order, 0.166, 0.120, 3.43, 0.02. With \( n = 0 \), (which he found for \( D_{50} \) by area replacing \( D_{90} \) by area), an allowance made for some of the data having different \( D \) systems, and \( D_{90} \) replaced by median diameter by weight estimated from some of the particle size distributions, the author found coefficients, 0.281, 0.125, 4.5, 0.17. These last values yield \( F_{bo} = 38 \cdot (D/d)^{1/2} \) for Fig. 10.4.

10.48. Note that Equations 10.16-18, with their appropriate coefficients can be transformed to contain \( b \), merely by writing \( 1.8^2Q^2/b^2 \) or \( 1.8^2q^2 \) instead of \( Q \); this is because, by Eq. 10.15 in which the Kellerhals value of \( X \) was 1.8, \( b^2 = 1.8^2Q \). The transformed equations are then true for all values of \( X \). They become:

\[ d \propto D^{-1/5} - 2n/5 \cdot Q^{1/5} b^{-4/5} \quad \text{(10.16')}. \]
\[ S \propto D^{0/5 + 7n/5} \cdot Q^{-4/5} b^{4/5} \quad \text{(10.17')}. \]
\[ V \propto D^{1/5 + 2n/5} \cdot Q^{-1/5} b^{-1/5} \quad \text{(10.18')}. \]

10.49. **Line or band in Fig. 10.4?** Eq. 10.20 states that if \( n = 0 \), the scatter band around \( F_{bo} = 38 \cdot (D/d)^{1/2} \) in Fig. 10.4, or around a more accurate replacement, does not imply that different parallel lines should exist for different \( D \)'s. If \( n \) is negative, as in Kellerhals' specific equations, then parallel lines of large \( D \) have low position, and vice-versa; so lines coming up from the regime theory phase would have to cross each other. There seems nothing physically impossible in this; in fact, with roughness increasing with \( D \) in the flat bed phase, and doing the reverse in the duned bed phase, perhaps crossing should be expected. Para. 10.24 suggests that lines should not cross, so implies that \( n \) is positive. Facts to settle the matter are not yet available.

10.50. **Comparison of \( Q \) formulas with regime theory ones.** Equations 10.16', 10.17 and 10.18 are comparable with Eqs. 7.9, 7.10', and 7.13. \( D \) is used in the former as \( F_{bo} \) contains \( d \); there is no equivalent of \( F_s \) in Eqs. 10.17,18 since all channels are imagined to run to fixed \( X \). In this system of fixed \( X \) and small \( d/D \), compared with a system (from one headworks) of regime theory canals, depth varies more rapidly with discharge and \( S \) tremendously more rapidly; thus the headwaters of a gravel river (corresponding to the small distributaries of the canal system) would be relatively steep.
If the headwaters had coarser material than the main river the steepness would be greatly exaggerated since it is proportional to $D^{6/2}$ instead of to about $D^{3/12}$ in sand canals or $D^{5/24}$ in canals with near-gravel sizes and large $d/D$. Velocities would be even more insensitive to discharge than in the canals, but more sensitive to change in $D$.

10.51. **Importance of graphed information.** The total phase picture is too complex, and information is too disjoined, for the use of formulas without guidance from plotted data. The quantitative and qualitative information from world flume data can be appreciated from the original Cooper and Peterson plots typified by Fig. 10.6,7. The Colorado State University work on phase limits in sand is condensed in Fig. 10.8, acknowledged to Simons & Richardson, 1966, and is presented alternatively in Fig. 10.6,7. Information from rivers with large and small gravel is outlined in Fig. 10.4 along with the related information of sand canals, all with small charges. Fig. 3.3 condenses information from sand rivers and very large gravel ones with ordinary bed-load charges. The complete set, taken in context, should prevent misapplication of formulas and advertise uncharted areas of knowledge.

**MISCELLANEOUS**

10.52. **Use of side-factor.** A curious detail that arises from Eq. 10.15 is that, in regime theory, the constant $X$ represents a constant ratio of $F_b$ to $F_s$ when sides are hydraulically smooth. In gravel rivers $F_b$ may correspond to a different phase of bed behaviour, and sides are not smooth. The reason the ratio persists, apparently as a measure of relative erodibility of sides and bed (in straight reaches of a river), is not clear. Because it does, there seems no harm in using a “working” side factor, $V^3/b$, for some stage where erosion is about to start on the banks. The author found that some similar large gravel rivers in Western Canada seemed to work to a side-factor, at a dominant discharge, given by

$$F_s = F_{bos}^8$$

where $F_{bos}$ means the zero bed-factor of a channel with bed made of the gravel sides under reference. Other rivers have been found to work to a different relation. No particular physical significance attaches to such rules of thumb, but they are useful in design.

10.53. **The fourth independent regime equation.** The fourth degree of freedom (para. 4.17) that rivers possess and canals do not is
meandering. This very systematic phenomenon was described in Chapter 2, definitions of quantities associated with it are in paras. 2.12, 13, and the history of quantitative studies leading to:

\[ M_L = 10b \] (3.1,10.22)

is in para. 3.2. One might speculate that the erosive actions that carve out an incised channel are exactly those that cause meandering, and that the great range of variables over which the equation has been established as a good average fit may indicate that it expresses a physical law; however, there is no obvious linkage with dynamical statements so the equation is not accepted as basic like the three basic canal relations. It will be accepted here as the tentative fourth independent regime relation peculiar to the property of meandering. Meander breadth \( M_{m} \), which is far more variable than \( M_{L} \), will be considered, without a special equation, in terms of a meander ratio \( M_{m}/M_{L} \) that has different values for different patterns of meandering.

10.54. Representative versus equilibrium discharge. In previous chapters the difficulty of estimating an equilibrium (dominant, formative or regime) discharge (para. 4.15) of a canal has been avoided by confining attention to cases where (a) it must obviously lie between very close limits, (b) slow variation of discharge permits every discharge at which there is active bed to be treated as an equilibrium one associated with its proper bed-load charge. The difficulty was also circumvented by using the regime theory slope formula containing the twelfth root of \( Q \) so that all reasonable assessments of equilibrium \( Q \) gave practically the same twelfth root. With rivers this slope equation, where applicable, still gives good, but not unlimited, service in looking after guesses at equilibrium discharge (between discharge at which a river bed becomes active and peak flood), but its need for a guessed meander correction coefficient, \( k \) (para. 10.36), remains. Also matters such as hydraulically rough sides, effect of suspended load on \( F \), and the poor accuracy of assessment of \( F_{10} \) in gravel rivers, all call for some kind of amendment of equations. On top of these matters, are the facts that the world’s good discharge data are confined to a relatively few sites on a few rivers, that a good assessment of equilibrium discharge for a wide range of active-bed discharges has to be based on a long hydrograph record, and that valuable data on regime quantities may exist without corresponding good discharge data. Consequently regime theory and non-regime-theory cor-
relations of river quantities against discharge will be found to use “representative discharges”, in the sense of ones that are standard­ized in some way but do not aim at being equilibrium ones; for example bankful, or annual mean flows may be used. In many circumstances such representative discharges are just as good as relatively accurately assessed equilibrium ones. For example, in a hydrologically homogeneous zone, most of the streams would have very similar hydrographs and probably even more similar annual peak flood frequency curves. So any discharge statistic in the range of active bed would probably bear about the same ratio to any standard one in each of the rivers; thus the flood of 20 years exceedance would be about the same number of times the median in all. Similiarly any statistic would be a fairly constant multiple of the equilibrium flow. The Leopold & Maddock analyses of river breadths and depths (paras. 3.34, 11.11-14) illustrate, for they were tried with various discharge statistics and gave practically the same fitting line slope for all. In summary, if regime data are analysed against representative discharges they should plot generally, on double-log paper, so as to be almost identical with the plots using perfect equilibrium discharges, but will be displaced bodily; stated alternatively, they will show the same functional forms but the coefficients will be different. For many practical purposes the functional form suffices.

10.55. **Bed-form measures.** Bed-form measurements have received only passing reference. A principal reason is that data are in­adequate to make use of Eqs. 10.2d, e. With sonic sounders now a commonplace, and mathematicians devising refined measures for wave parameters (Nordin, 1966), the way is open for these equations to receive as much attention as Eqs. 10.2a, b. Because λ and α can be used to eliminate two awkward variables from the d,S relations—for example C which is not measurable in the field and a major X . . . factor such as gravel dispersion—study of wave relations might give simple formulas that would be of great practical use if wave parameters are readily observable in the field.

10.56. **Different viewpoints.** For brevity and unity the present chapter has taken, mainly, the viewpoint of the engineer who sees channel dimensions forming themselves to suit the sediment com­plex discharge imposed by Nature or himself; bed-forms have been seen as concomitants of the adjustments. Another viewpoint can start from the qualitative properties of bed-forms followed by inquiry about the relations among channel parameters associated
with them; and the inquiry may be pursued in flumes where the channel dimension, S, is imposed and, with a given flow, causes a sediment load to be entrained to form the complex. Mathematically the only difference is an interchange of independent and dependent variables in Eqs. 10.2. Physically, mastering this alternative method is most desirable as an aid to “understanding” and to skill in solving practical problems. References have been chosen to direct attention to it. Other alternatives are possible, but a complete one must deal with the laws relating to the degrees of freedom of the problem.
CHAPTER 11

APPLICATIONS OF REGIME FORMULAS TO RIVERS

11.1. Introduction. A working postulate will be adopted from para. 10.35 as: "For the proper phases and conditions, steady flow regime equations can be applied, with different coefficients for different cases, to equilibrium problems of rivers". So this chapter will make and discuss selected applications, of practical interest to engineers and geomorphologists, in a graded sequence. The first group will be plots of very general river data to coordinates inspired by, or closely related to, regime formulas; they may be regarded as methods of classifying river data for ready and instructive quantitative use or as tests of the postulate, as the reader wishes. The second will be applications to important practical problems in which the formulas are of major assistance. The third will comprise various problems that must be solved somehow by the engineer with little theoretical aid of any kind; some may not be regime ones at all, but are associated with regime conditions in the river. The selection is for practical value and to provoke thought and argument. All problems are related to the real-life kind where the engineer, as usual, has to form his own judgement in terms of a few physical principles, a few exact solutions of highly idealized cases, and his own experience and general knowledge, to suit real cases where indeterminate factors abound and may even predominate. Therefore the chapter should be read with the attitude of mind that would be accorded to a report to a large engineering organization by a specialist; if all the answers were known the specialist would not exist.

RELATIONS AMONG RIVER VARIABLES:
QUANTITATIVE CLASSIFICATION

11.2 Tests of regime slope formula. Potentially the most powerful formula to test against observed data is the regime theory slope equation \( (7.11''') \) modified to:

\[
S = k \cdot \frac{F_{bo}^{11/12}}{Kb^{1/6}Q^{1/12}} f'''(C) \quad (11.1)
\]
in which $k$ is the meander correction coefficient explained in para 10.36. As discussed in para 8.5-8 its outstanding advantages are that slope is the steadiest regime quantity and is exceedingly insensitive to variations in breadth and, particularly, discharge. However, it has not been used until recently because of previous inability to assess $F_{bo}$ for gravels (para 10.44). Blench & Qureshi's, 1964 graph of the equation with trial data is reproduced as Fig. 3.3, in the form of:

\[
Z = \frac{b^{1/6}S}{F_{bo}^{11/12}} \quad \text{against} \quad \frac{k \cdot f''''(C)}{KQ^{1/12}}
\]

It is intended for use as a Regime Slope Analysis Chart (para 11.6) with $F_{bo}$ given by Fig. 7.3. The bottom line of the graph, sloping at minus $1/12$, is for $k = 1.0$, $f''''(C) = 1.0$, so represents straight canals of vanishingly small bed-load; $K$ was fixed at 1,920 corresponding to 50°F., which is a fair average for the annual behaviour of all rivers. The intermediate line, for $k = 2.0$, $f''''(C) = 1.0$, represents such canals that have been neglected long enough to acquire marked meandering. The top line represents the meandering canals given a bed-load charge of about 9, so that $f''''(C) = 2.0$; this was selected because experience has shown that only a few rivers have larger $f''''(C)$—though some exceedingly heavily laden sand ones have charges of the order of 100, implying $f''''(C)$ about 7.0.

11.3 Probability of fit of data to Fig. 3.3. Before plotting real data on Fig. 3.3 to test the validity of equation (11.1) there is advantage in estimating causes for points falling outside the full band, whose vertical breadth is log 4.0 corresponding to a variation of 4 times in $Z$. Assuming first that $S$, $b$, $D$ (for determining $F_{bo}$), and the equilibrium value of $Q$ are measured perfectly, and that variation of $K$ from truth is negligible, then para 11.2 says, virtually, that no point can fall below the band and those that fall above have $kf''''(C)$ greater than 4.0; but the streams must be in the phase proper for use of the regime formulas. However, if gravel river data in the non-regime-theory phase for $F_{bo}$ (Fig. 10.4) are plotted using Eq 10.12, the rough information in Chapter 10 suggests that they will be displaced upwards about half a band-width for a 30,000 cusec stream with 4 inch gravel and small discharge, and progressively more for smaller streams with the same gravel, till they are out of the band by 3,000 cusecs and well out of it when $Q$ is down to a couple of hundred; at this lower value Eq. 11.1 has been
superseded by the completely different Eq. 10.17. Small sand models of negligible $C$ will probably plot above the band.

11.4 Unavoidable errors of measurements and assessments, as distinct from mistakes, seem unimportant. A glance at Fig. 3.3 shows that, even were $Q$ assessed 10 times wrong very little change in vertical position of its point could occur; similarly, error in $b$ is unimportant; it is hard to imagine $F_{\text{m}}$ being assessed as much as 25% wrong, and this would give an error ratio of about 1.25 compared with the band width of 4 times.

11.5. **Mistakes and misunderstandings in data collection.** A major cause of points plotting above or below the band is inexpert data collection. This situation occurs because quantitative fluviology with a dynamical background is a very new science. Records to date may not show whether a river bed is sand or gravel; size of bed-material is rarely measured; slopes are stated without indication whether they are down the valley or along the flow, or for short discharge reaches or for many meander bends, or for backwater or draw-down due to natural, hydroelectric or beaver dams; slopes may include rapids and waterfalls without mentioning the fact; reported discharges may be annual means far below formative; beds may be stated as gravel or sand and turn out to be of large rigid boulders with pockets of mobile material interspersed, or to be fighting their way through fallen trees and clotted vegetation. Thus, utterly wrong assessments of regime slope must be expected in all data collections to date and can cause errors of fully a band width in either direction. Therefore, for future slope analysis, data must be collected under rigorous engineering control. The data plotted in Fig. 3.3 were accepted only because their histories indicated that they were free from major mistakes; also they were all for supposedly small or normal bed-load charges. Neill, 1965 (a, b), and 1967 (a) advises on and exemplifies good data collection.

11.6. **Data analysis by Regime Slope Analysis Chart, Fig. 3.3.** The inverse use of the regime slope chart, once its validity is established, is to classify river regime behaviour of all the streams that come under the responsibility of a Water Resources organization. Its function is then comparable with that of a set of flood frequency analyses which convey vast information to an expert hydrologist.

11.7. **Analogy with Specific Speed.** The reader acquainted with pump and turbine selection will note the parallelism between Fig. 3.3 and a chart of Specific Speed against a dimensionless design
quantity for indicating type of design. Such a Chart is derived by stating a "prototype case" (para 10.12) and reducing it to equations like Eqs. 10.2. The major independent numeric is devised to contain the three most important quantities including rotational speed, $N$; after being stripped of the arithmetic nuisances $\rho$, $g$ it is called (loosely) the "specific speed", but is neither a speed nor a numeric. Then a couple of unimportant numerics are rejected, leaving two for turbines. Finally it is agreed to confine attention to a specific condition, namely maximum efficiency and this removes one more numeric. Thus, for one design, there is one value of specific speed and, therefore, of everything else that is dimensionless. A graph of specific speed (at maximum efficiency) against, say, cavitation number for different designs could then have design types noted along its length. Fig. 3.3 is more general because the "Specific Slope", $Z$, against $Q$ (imagined non-dimensionalized by using $r$, $g$) plot gives, for a certain $Q$, points whose vertical position tells the "design" of the river in terms of charge, or crookedness, or some combination. In use, the position of every plotted point of a regime slope chart, even within a half-band, directs attention to matters meriting investigation by field inspection and measurement. Characteristics found in the field are recorded in each river's history along with which a copy of the chart for all rivers and the various hydrologic summaries of the particular one are filed. Thus all the information required for assessing the consequences of any kind of engineering interference with every river comes on record in condensed and efficient form. During initiation of use of the chart there is advantage in plotting any available slope data of rivers regardless of whether the information is likely to be subject to the errors discussed in para 11.5. The positions of the plotted points then indicate possible errors and genuine regime peculiarities; when the former have been eliminated by resurvey the final chart shows only regime peculiarities. For medium and small gravel rivers a chart based on Eq. 10.17', with $q$ replacing $Q/b$, should be devised.

11.8. Correlation of $S$ and $Q$. Geomorphologists are interested in the statistical relation between river slopes and representative discharges as they occur in nature. A major collection of data from all sorts of rivers, including bed-material sizes when available, has been made and recorded by Leopold and Wolman, 1957, with discharges quoted as estimated bankfull where possible; the $S$, $Q$ data were reduced to a graph on which the scatter was in a band-width
of about 100 times. Geologically such a correlation is valuable in showing how nature associates $S$ with $Q$, but, except by luck, it cannot show how $S$ is linked to $Q$ by a dynamical law. The reason is that $S$ depends on $Q$ in a manner laid down by dynamical law and, in addition, on bed-material size and charge. The last two, in nature, seem likely to be larger on an average in the small rivers: of a large random sample of a system than in large rivers; therefore $S$, in smaller rivers of a system, is likely to be larger than indicated solely by the index of $Q$ in the proper dynamical formula. Specifically, the trend line on an $S$, $Q$ chart should be expected to have a numerically greater slope than $1/6$ where Eq. 7.10 applies, and greater than $2/5$ where Eq. 10.17 applies.

11.9. The order of the amount by which plotted data will scatter on an $S$ against $Q$ chart, as compared with Fig. 3.3, can be estimated for the regime-theory phase. Such a plot is comparable to preparing Fig. 3.3 with $b$ and $F_{bo}$ wrongly assumed constant and without Fig. 3.3’s precautions to avoid channels with bed-load charges likely to exceed 9 and to exclude data that had not been collected with engineering care. Now, $b^{1/6}$ is a roughly constant number of times $Q^{1/12}$, so its omission will not affect scatter much, but it will make the fitting line likely to slope at minus $\frac{1}{6}$ instead of minus $\frac{1}{12}$. $F_{bo}$ varies from about 0.5 for fine sands to 10 for large gravel (in a very large river), so omission of $F_{bo}^{11/12}$ should produce scatter within a band of about 17 times range. If we accept $f''''(C)$ for an exceedingly heavily-laden river as about 7.0 against the top limit of 2.0 for Fig. 3.3, a further band width of $7/2 = 3.5$ times range is introduced. So, if one assumed the worst possible combination for the factors discussed so far, a band width of $4 \times 17 \times 3.5 = 240$ times could be expected. The odds against everything being as extreme as possible are very high, so the scatter of 100 times quoted in para 11.8 seems reasonable, even allowing for some of the data probably having very wrong slopes and being out of phase.

11.10. The $S$, $Q$ data of the reference in para 11.8 were fitted by a line sloping at minus 0.44, though considerably different slopes exceeding $1/6$ would also fit the very scattered points well within the limits of statistical significance. Therefore, in terms of the preceding regime theory type of discussion, and, allowing for a mixture of phases fitted by the minus $1/6$ and minus $2/5$ powers of $Q$ respectively, there are grounds for believing that small channels of a system are associated with larger bed material.
11.11. **Sectional analysis of river regime.** Leopold and Maddock, 1953, initiated quantitative analyses of breadth and depth of rivers by plotting, for gaging sites, the relations between b and Q and d and Q within one river system (main stem and tributaries) at a time. Q was a representative discharge (para 10.54) accepted, after trying various others, as the annual means discharge; b, d corresponded to this discharge. Their procedure, therefore, was essentially the same as the initial one of Lacey, 1929, on canals and was probably forced by the same two circumstances, viz relative lack of slope data, particularly within one system, and appreciation of the need to hold what we now call $F_b$ and $F_s$ as constant as possible since they are not directly measurable. Fig. 3.2 taken from their work condenses results for systems in different physiographic settings and includes a line for Madras (India) irrigation canals. An interesting verification of the utility of the representative discharge concept was that they used discharges of various frequencies and found that all gave about the same slopes of lines as on Fig. 3.2. They used surface breadth at the representative discharge for b, and the corresponding mean depth for d.

11.12. From a regime viewpoint Fig. 3.2 is a chart aimed at testing the basic equations (in their practical form):

$$ b = \sqrt{\frac{F_b Q}{F_s}} \quad (7.8, 11.3) $$

$$ d = \sqrt[3]{\frac{F_s Q}{F_b^2}} \quad (7.9, 11.4) $$

in which $F_b$ and $F_s$ are defined in terms of surface breadth and mean depth of approximately trapezoidal sections. The test is, properly, at a presumed definite fraction of true equilibrium discharge in each system. From what is known now, as a result of slope analysis (para 11.8, 9), about the association of large $F_b$ with small Q, or even from the general knowledge that small distributaries are likely to be rather heavily laden and to possess coarser bed-material on an average than the main stems, one should not expect to find the indices of Q in equations (11.3, 4) confirmed too well.

11.13. In fact, confirmation of the index $1/2$ for Q in equation (11.3) is astoundingly good; it is Leopold and Maddock's exact
average. An explanation could be that the equations are mathematical identities that possess meaning only because, in any phase, $F_b$ and $F_s$ are measures associated with erosive attack on bed and sides; and system channels with coarser bed material normally have coarser sides, so that $(F_b/F_s)^{1/2}$ cannot vary much. However, the near-unanimity about the index $1/2$ among a variety of field investigators is so remarkable, in a controversial subject like hydraulics, that the true explanation may be dynamical—for example in the relation between side and bed shear stresses (Rajaratnam, 1968).

11.14. With $F_b/F_s$ fixed, by Nature, in a system (on an average), Eq. 11.4 for regime theory becomes $d \propto (Q/F_b)^{1/3}$ and Eq. 10.16 for flat gravel bed is $d \propto (Q^2/D)^{1/5}$. So, in a system with fixed $F_b$ or $D$, but with channels in both phases, the relation of $d$ to $Q$ should be according to a power of $D$ between $1/3$ and $2/5$. The Leopold and Maddock figure averaged exactly 0.40 so, again, there is a likelihood that small channels of a system have larger bed material and possibly larger $C$, on an average.

11.15. The preceding paragraphs outline the major tests of validity of the three basic regime theory equations and, less directly, of the corresponding formulas for subcritical moving flat bed, in terms of field data. The following will demonstrate the practical utility of the regime theory ones since experience with them is extensive; adjustment of the findings to suit formulas for other phases is mainly verbal and is left to the reader.

**MAJOR APPLICATIONS TO ENGINEERING PROBLEMS**

11.16. **Interference with river regime.** A principal section of practical problems reduces to “Why were certain human interferences with a river followed by certain consequences?” and, if foresight is going to be exercised, “What are the likely quantitative consequences of the following proposed interferences?” Interference can be viewed in terms of alterations to the major independent variables $F_b$, $F_s$, $Q$, $k$—in which $F_b$ can be altered through $F_{bo}$ and $C$—which influence four major dependent variables $b$, $d$, $S$, $M_r$; factors such as special effects of suspended load are relevant but cannot be covered exactly by equations yet. So, however the matter is viewed, it cannot be simpler, in general, than explaining quantitatively, by formulas that are available, how each of four principal factors is affected by variations in five others.
Obviously, answers given without the aid of four formulas required by the four degrees of freedom (para 4.17) are liable to serious errors and omissions. Equally obviously the classification and discussion of all combinations of problems soluble by the equations would be tedious, particularly if the effects of secondary factors not explicit in the equations were discussed as they should be. Therefore the following problems are not exhaustive; they are intended to illustrate enough to make all others soluble with practical accuracy. Table 11.1 is presented to summarize the different “directions” in which the major independent variables affect the dependent ones.

**Table 11.1.** Signs of dependent variable changes for increases in independent variables.

<table>
<thead>
<tr>
<th>b</th>
<th>d</th>
<th>S</th>
<th>M_L</th>
</tr>
</thead>
<tbody>
<tr>
<td>+</td>
<td>-</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>-</td>
<td>+</td>
<td>small</td>
<td>+</td>
</tr>
<tr>
<td>+</td>
<td>+</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
</tbody>
</table>

11.17. **Sectional changes due to changing F_b, F_s.** Good fluvio logical data are seldom available (para 11.5), so, to preserve practicality, the following two problems are stated with roughly the limited amount of data available in the prototypes on which they are based. They concern the effects of interfering with sediment load.

**Problem 11.1.** A river with sand bed, large suspended load of all smaller sizes, a sand-loam flood plain, and a definite incised channel (though it was subject to spill) was dammed so that all the load was trapped. A gaging station many miles downstream of the dam showed a marked permanent change of sectional form a few years after the dam had been built, as compared with the section just at the time of building, but, so far as could be determined, long before the effect of cutting off bed-load could reach it. The changes at bankfull condition of the incised channel amounted to the time-mean breadth, depth and mean velocity changing to 0.85, 1.80 and 0.67 times their old quoted values. What does this show about F_b and F_s?

**Answer.** By equations (7.1, 2) F_b and F_s, estimated at bankfull conditions, had altered to $0.67^2/1.80 = 25\%$ approx. and $0.67^3/0.85 = 35\%$ approx. of their original values.
11.18. Inquiry about the remarkable drop in bed-factor showed that it could be explained by the effects of local diversion works wearing off; the final absolute value corresponded reasonably with the bed-material size and very small bed-load charge. The drop in working side-factor had to be expected in a channel that can build berms (para 5.12) during medium supplies and erode them in high ones, for removal of the suspended load removes this self-repairing ability. Incidentally, the side-building property is one reason why side-factor is not discussed in terms of such a simple concept as “shear stress”, but is referred to “erodibility” and “depositability” (para 7.7). The drop of side-factor corresponds to the rough design range of 0.3 to 0.1 (para 7.18) between very cohesive and very sandy loam banks.

11.19. **Need to combine equations with field evidence.** The problem illustrates how misleading a qualitative opinion might be. This channel actually tightened after removal of suspended-load, so might have been quoted to “prove”, in a law suit, that removal of suspended load cannot cause a river to widen. The reason it did not widen was that $F_b$ and $F_s$ had dropped together; only a pair of proper equations can show these effects. On the other hand, only site information could reveal why the bed-factor changed; without that information one might be led to a wrong conclusion (on the presented evidence alone) that this river had a very large natural bed-load charge originally. In fact, formulas will analyse evidence quantitatively, within their range of applicability; but their answers are not a substitute for field evidence.

11.20. An illustration of the need to use all the regime equations if the data are available is the following problem.

**Problem 11.2.** A river, similar to that of Problem 11.1, gradually increased its breadth to about 1.8 times and decreased its depth by “several feet” in a reach of many miles, downstream of long-term soil-conservation activities that were aimed at, and claimed to have succeeded in, reducing sediment load. There were no sediment-size observations, but residents had not noticed any change in bed-material size. There was no complaint or evidence of specific gages (para 4.5) having risen. Popular opinion was that, somehow or other, “extra sediment” had got into the river despite soil conservation efforts. Discuss this opinion in terms of $F_b$, $F_s$.

**Answer.** By equation (7.8), if change of $F_b$ was the sole cause then $F_b$ should have increased about $1.8^2 = 3.25$ times and, in
order to move the extra load, the regime slope would have increased, by equation (7.11'), about 3.25^{5/6} = 2.7 times, assuming that banks adjusted to keep $F_s$ the same. This increased slope would have been accompanied by floodings and depositions of bed-material on the flood plain that could not have escaped notice. If change of $F_s$ alone were the cause then $F_s$, by equation (7.8), would have dropped to about $1/1.8^2 = 0.30$ of its original value which is within normal expectation; $F_s$ has practically no effect on regime slope. By equation (7.9) the depth would tend to drop to $(0.30)^{1/3} = 0.67$ of its original value. Inquiry about the nature of the conservation actions showed that a major one was a change of cropping practices to stop sheet-erosion, and this would tend to remove the cohesive material from the suspended load of the river.

11.21. As in para 11.19 a warning is needed that equations are not a substitute for total physical knowledge. A peculiarity about suspended load is that it travels at about water speed, so its effects can be felt relatively early throughout a system. The effects of changes of bed-load may travel through a sand-bed system at a speed of the order of a mile a year; therefore a few decades may elapse before they start to act at a remote site. Again, if sides start to collapse because of a drop in suspended load, the collapsing material adds to total load, and how this affects the rate of development of the overall problem depends on what the sides contain—some of the material will go to raising the bed and some will travel in suspension and may or may not be cohesive; in general, the effect of a reduction of suspended load at an upstream point will not act instantaneously with full effect everywhere downstream but should act far more rapidly than a change of bed-load.

11.22. Special cause of increased river breadth. Greatly increased river breadth has been noticed upstream of a new reservoir whose long-term level fluctuations of level were somewhat more than the normal range of fluctuation of the river; surveys showed that the increase worked progressively upstream from the time the reservoir was put into operation. Here the cause appears obvious enough from field observation but would hardly be deducible from regime formulas or Table 11.1 without knowing the circumstances. As a delta advances into a reservoir and buries the old river channel a new high-level river forms through it. The water-levels in this river, near the delta end, may be regarded as trying to fluctuate to suit the discharge, approximately as they do a long way upstream;
but the lake goes up and down to suit the floods that fill it and the
human demands for irrigation, power, etc., that empty it. If a flood
arrives after a drought which has depleted the reservoir there may
be almost a waterfall over the end of the delta into the reservoir
and violent erosion will occur in that zone; Bondurant, 1955, has
reported on this occurrence, with photos. The backwater effect
from the large draw-down produces abnormally high velocities far
upstream, increases $V^2/b$ and causes banks to rip out. If the
river’s load is not the kind that rebuilds banks readily the
enhanced breadth may remain indefinitely, although, when the
reservoir refills, the bed that was scoured, along with the banks,
will be rebuilt from the coarse part of total load. In this case the
regime equations still apply, but $F_s$ corresponds to the exaggerated
velocity due to drawn-down and not to the velocity under average
conditions, or under those that will obtain when the delta has
advanced so far that the original site is well inland. Channel
broadening, due to backwater, might occur also upstream of where
engineers have eliminated a couple of river bends, or even up­
stream of a natural cutoff if a very large flood arrived shortly after
the occurrence—that is, before the river had managed to readjust
slope locally (para 2.24).

11.23. **Enhanced meander activity.** Farmers complain frequent­ly that engineering or other interference with a river causes
meander activity to increase and to damage their land, or make
normal river problems occur more frequently. According to
equation (3.1), any actions that increase regime breadth of the
incised channel should be expected to tend to increase meander
length; and, if the meander ratio has no cause to alter, the meander
breadth must increase in proportion. Such complaints would grow
over a period covering several large floods—perhaps a couple of
decades.

11.24. **Design of an assisted cut-off.** The problem of the assisted
cutoff (Plate 7) has received considerable attention in paras 9.6
et seq. Algebraically it is also the problem of interfering with a
river by trying to straighten it, as illustrated in para 3.11, Problem
3.1. A simple standard method of solution is to write whatever
slope equation is applicable, insert the quantities for the “dis­
turbed river”, then insert the quantities for the “undisturbed river”,
and divide the one equation by the other. To illustrate from
equation (11.1), let suffix u mean “undisturbed” and suffix d mean
“disturbed”. Then one obtains:

114
\[
\frac{S_d}{S_u} = \left[ \frac{k_d}{k_u} \right] \cdot \left[ \frac{F_{bd}}{F_{bou}} \right]^{11/12} \cdot \left[ \frac{f'''}{(C)_d} \right] \cdot \left[ \frac{b_d}{b_u} \right]^{1/6} \cdot \left[ \frac{Q_u}{Q_d} \right]^{1/12}
\]
(11.5)
in which \(K\) has been cancelled out. In this equation every dimension of the one case appears as a multiple of the corresponding one in the other, which is a convenience when a set of numerical problems has to be solved and the answers have to be compared. Writing can be saved by making the convention that, when the letter \(R\) (for ratio) is written alongside an equation number then every term in the equation means "the ratio of the new condition's dimension to the old". This permits equation (11.5) to be written as:

\[
S = k \cdot \frac{F_{bo}^{11/12}}{b^{1/6}Q^{1/12}} \cdot f''(C) \quad \text{(11.6R)}
\]
and shows that a ratio equation is exactly the original from which it was deduced, with any common factor (such as \(K\)) omitted.

**Problem 11.3.** An exaggerated river loop is to be provided with a straight assisted cut-off tangent to the outside of two adjacent bends. The loop to chord ratio is 3.5. The designer would like to open the cut at the stage where the river is just moving its bed (so that \(f'(C) = 1.0\)) and decides to design the cut so that, at this stage, its \(f'(C)\) will be 3.0 and its side-factor 4 times that of the river. He assumes the meander correction coefficient of the bend, for using the slope formulas, is 2.0.

(i) For what percentage of the river discharge, at the opening stage, would he design the cut?

(ii) What fractions of loop breadth and depth at that stage should he give the cut?

**Answer.** Note that the \(S\) ratio is 3.5. Then applying equation (7.11') multiplied by \(k\):

\[
3.5 = \left( \frac{1}{2} \right) \cdot \left( 1.0 \right)^{5/6} \cdot \left( 4.0 \right)^{1/12} \cdot \left( 3.0 \right) \cdot Q^{-1/6} \ldots \ldots (7.11'\text{R}) \text{whence } Q = 1/80 \text{ approx.} \]

Now, for \(f'(C) = 3.0\), Fig. (7.2) gives \(C = 28\) approx., so \(F_{bd} = F_{bo} (1+3.36)\) by equation (7.5) and \(F_b = 4.36\) \ldots \ldots (R). Therefore \(b^2 = (4.36/4.0) \cdot (1/80)\); \(b = 0.12\) \ldots \ldots (R) approx., and \(d^3 = (4.0/4.36^2) \cdot (1/80)\); \(d = 0.14\) \ldots \ldots (R).

This kind of solution draws attention to the very great effect of enhanced slope, or, what is the same thing, loop to chord ratio, on the regime discharge of a cutoff channel. Without further calculation it can be seen that, in the case of a sinusoidal channel
(Plate 1) about to cut off, if we take \( k = 2.0 \) and the loop chord at 1.5 for cut-off along the edge of the meander belt, then the regime discharge of the cutoff channel is \( (3.0)^{-1/6} = 1/7,000 \) approximately of the river discharge. It is not surprising, therefore, that such meanders, associated with spill that can find short-cut paths, do not develop loop-chord ratios greater than about 1.5.

11.25. **Straightening a river.** Engineers are impelled to consider straightening rivers for reasons such as simplifying highway location, reducing number of bridges, improving navigation, and lowering flood levels. Circumstances may permit such actions to be economic in terms of the consequences to the river, especially if the thickness of mobile bed-material is small. The effect on the river, if it remains of regime type, is bound to be violent bed-erosion in the straightened reach, initial bed-erosion and degradation upstream and initial bed-deposition and rise of specific gages downstream. Thereafter, if the straightness is maintained indefinitely the whole river to the first control point upstream will degrade to suit the ultimate inevitable degradation at the head of the straightened reach and the raised downstream reach will gradually return to normal; see also para 3.11. Equation (11.6R) gives a numerical idea of the action if \( S \) is made 1.5, \( k \) is made 1/2 and the other terms except \( f''''(C) \) are left intact. Then \( f''''(C) \) increases in the ratio 3.0 which means that \( C \) could increase, initially, from about 1 to 20 or from 10 to 80 (scaling roughly from Fig. 7.2); in a river like that of Plate 4 the increase in \( S \) would be about 2.5 times.

11.26. **Estimating Bed-Load Charge.** In a few practical problems knowledge of bed-load is important—for example in assessing the dredging task if a port is to be built, or the discharge at which an excavation across a river will fill too fast to permit construction. However, a considerable engineering literature is devoted to formulas purporting to permit the calculation of bed-load from data of laboratory flumes, and theories are advanced to justify the formulas. From Chapter 10 we known that world flume experiments, as a body, suffer from ignoring the existence of phases, failure to consider all the variables, association among the variables, use of non-standard alleged bed-material, confinement to one degree of freedom, trifling range of median particle sizes, and disagreement about the definition of bed-load. Moreover, speculative formulas tend to be based on rigid-boundary fluid mechanics, which is not yet perfect, and to ignore the possibility that a water-sediment complex and a mobile bed might produce some novel phenomena.
Finally, even correct formulas for flumes would not give reliable results under the complex conditions (para 10.31 et seq) of many rivers. It is hardly surprising that popular formulas, applied to estimating the bed-load for a specific ideal case, show a range of answers, in Fig. 9.1, with the largest about 100 times the smallest. Engineers have tolerated such wild contradictions probably because they seldom have practical use for a figure of bed-load charge even if it were correct; moreover there is no means for measuring bed-load in the field as a routine. Pending improved research, fair solutions to some problems can be obtained by seeking and studying relevant individual sets of data. For such study Simons and Richardson, 1966, and Cooper and Peterson, 1968, coordinate most qualitative and quantitative laboratory information and appreciable field information; in addition, Einstein, 1944, and Colby and Hembree, 1955, record scientific observations of bed-load under field conditions. Laboratory data may have unrealistic b/d ratios. Chapter 10 directs attention to major points.

11.27. Sensitivity of Transport Formulas. An unavoidable weakness of transport formulas used to calculate C is their sensitivity to errors in the quantities inserted in them. The point is illustrated by Eq. 7.10" where, if \(1 + C/233\) is calculated 5% wrong, due to such errors, and if the true value of C is 5, the error in C is 240%. The bed-factor type of formula, Eq. 7.5, is less sensitive because the coefficient of C is larger, but it is most unreliable for C greater than about 20. In fact computing C from a formula is comparable, for inaccuracy, with calculating channel discharge from slope.

11.28. Algebra of Calculating C. The solution of Eq. 11.1 for C is awkward, but is simplified by Fig. 7.2, whose firm curves are used in the following problem for the sake of the exercise; para 10.27 comments on the dotted curve.

Problem 11.4. What is the bed-load at \(Q = 720 \text{ c.f.s}\) in a sand-bearing channel of breadth \(b = 120 \text{ ft.}\) with slope \(S = 1.29/1,000\) when water viscosity is \(1.21 \times 10^{-5}\); median sand size is \(0.28\text{ mm}\). The channel is in regime, and there is no suspended load. Meander correction coefficient \(k = 1.25\).

Answer. \(K = 3.63g/\nu^{1/4} = 1,980\); \(b^{1/6} = 2.24\); \(F_{bo}\) (by rough formula for medium sand) \(= 1.9\sqrt{0.283} = 1.01\); \(Q^{1/12} = 1.73\). Using equation (7.11"""), \(f''(C) = 7.94\). Fig. 7.2 gives \(C = 110\). Tons per day \(= 0.027CQ = 2,140\). Lbs/sec.-ft. \(= 2,140/120 \times 43.2 = 0.41\).
Graphic solution can be avoided by writing Eq. 11.1 as:

\[ f''(C) = \frac{(K/k) \cdot S \cdot b^{1/6} \cdot Q^{1/12}}{F_{bo}^{11/12}} \]  

(11.7)

But from \( C = 1 \) to \( C = 10 \), Fig. 7.2 is fitted well by \( f''(C) = 1.12C^{1/4} \), and from \( C = 10 \) to \( C = 200 \), \( f''(C) = 0.575C^{13/24} \). So approximately:

**For \( 1 < C < 10 \)**

\[ C = \left( \frac{1}{1.12} \right)^{1/4} \cdot \frac{(K/k)^{1/4} \cdot S^{1/3} \cdot b^{2/3} \cdot Q^{1/3} \cdot F_{bo}^{11/3}}{} \]  

and \( CQ \propto b^2 Q^{4/3} \)  

(11.8)

**For \( 10 < C < 200 \)**

\[ C = \left( \frac{1}{0.575} \right)^{24/13} \cdot \frac{(K/k)^{24/13} \cdot S^{24/13} \cdot b^{4/13} \cdot Q^{2/13} \cdot F_{bo}^{22/13}}{} \]  

(11.9)

and \( CQ \propto b^{10/13} Q^{15/13} \)  

(11.9a)

A striking feature of these equations is the sensitivity to \( k \), which transport equations have usually ignored. Apparently the use of slope over a complete meander bend, in a flume transport equation, could give a prophesy of 16 times the proper charge if \( k \) were neglected.

11.29. River Factors in Flume Transport Equations. The effects of some river factors on canal transport formulas can be seen by transferring Eq. 11.1 to Fig. 9.1 as lines marked “Regime”. These lines have slope 4/3 to agree with Eq. 11.8a. The one on the left-hand diagram is for \( k = 1.0 \) because the plotted points were calculated from the modified Einstein equation by inserting river dimensions of various stages and ignoring \( k \). (Texts sometimes state, loosely, that such a procedure gives an “observed bed-load”). The one on the right was given \( k = 1.25 \) as the channel was fairly straight. The curves are from formulas using overall river data and ignoring \( k \). Strictly, the Regime lines should allow for \( k \), in Eq. 11.8, decreasing as stage increases, since a high river “jumps its bends”; that is, they should be steeper. Again, river breadths normally increase with stage, and this calls for further steepening; if, for example, \( b \propto Q^{1/5} \) (Leopold and Maddock, 1953) the slope of 4/3 should be increased to 11/6 to suit Eq. 11.8. Finally, suspended load usually increases in intensity with stage and this would probably cause more steepening. In general, therefore, a transport formula derived from river observations, or adjusted to suit them, should be expected to plot steeper on a diagram like Fig. 9.1 than a flume-derived one.
PROBLEMS OF TRAINING WORKS

11.30. Obstructions and restrictions. As a river has a natural pattern and scale of behaviour a training work is, sooner or later, an obstruction or restriction. The more the natural tendency of the river is defied the more the work is likely to cost, so there is advantage in planning, if possible, to compromise with how the river prefers to behave. In any case design has to be in terms of the regime dimensions of the river whether the works are bridges, barrages, revetted banks, or spurs (groins) which are armoured points connected by embankments above flood level to high ground. The next paragraph illustrates the works and ordinary problems associated with a barrage on a spilling river as a preliminary to discussing each type separately.

11.31. As meanders progress downstream, cut off, redevelop, and so on, the presence of an obstruction like a bridge in a meandering river results sooner or later in the river folding, or "concertinaing" on to the bridge till relieved by a cut-off. Fig. 11.1 shows six barrages across Punjab rivers of from 400,000 to 1,000,000 cusecs peak floods and illustrates the point. The barrages may be looked on as bridges—they produce no ponding at high flood—and their right and left marginal banks (RMB, LMB) as roadways. They are provided with right and left guide banks (RGB, LGB) so that the collapse of river does not occur on to the marginal banks but, instead, on to pockets of water whose gentle circulation will not harm the banks. In case (a) a T-head spur has been built to straighten the river that had folded over to the dotted course marked 1931. In case (b) the river is temporarily split into two channels (an unstable condition) and both halves are collapsing on to the noses of the guide banks. Case (c) has good approach banks that refuse to yield, so is not typical. Case (d) is starting to show the effect at the right guide bank and has experienced it on the left where the channel has died. Case (e) could be typical of an exceedingly extreme occurrence, but the apparently collapsed course happens to be an old river one before the barrage was built. Case (f) shows spurs built to prevent the effect. Without spurs to prevent the effect, or guide banks to compel it to occur well upstream, the river must eventually fold on to the road or marginal banks.

11.32. Economic span for bridge or barrage. For a deeply incised river, with little or no spill, the span (of all bays) of a bridge has
to be approximately the natural waterway. For a widely spilling one, as in Plates 2, 14, the same reasons seem to dictate a span approximately equal to the breadth of an adjacent fairly straight deeply incised reach; spanning the whole flood plain, once common, gives virtually a series of bridges each of which comes under full attack as the river shifts in the flood plain. If the spilling river has no suitable incised reach then a regime theory solution of the span problem is to estimate what dimensions such an incised reach would have if the banks were of the largest bed-material, or of clay loam if the bed is of sand. The calculation consists merely of analysing the river by the slope formula, equation 11.1, to estimate its $F_b$ at high flood, and then finding the breadth of a regime canal with perpetual design flood discharge, the estimated $F_b$ and a suitable $F_s$. The breadth so obtained, or obtained by observation of an incised river reach, is called the flood breadth, $b_1$; if the observed reach has some spill its breadth, if spill were prevented, should be computed by multiplying the actual by the square root of the ratio of design flood to design flood minus spill.

11.33. The flood breadth forms a starting point for deciding on either bridge or barrage span, and experience shows that it is usually close to final acceptance. A longer span does not usually justify cheaper foundations, and a shorter one raises foundation and revetment costs while reducing superstructure.

11.34. Guide banks for bridges. Reasons for guide banks at a barrage have been given (para 11.31). For a spilling river the extra reason affecting a bridge is that, without a guide bank, a bridge abutment would act, periodically, as a spur nose of which the roadway was the shank; the scour at it would be large. The nose of a guide bank will receive as severe attack as would the abutment in its absence, see Plate 16 which demonstrates flow practically at right angles to a guide bank in a model. The cost of a guide bank is likely to be comparable with that of protecting the road in its absence, and the loss of a guide-bank nose in a very large flood is usually considered unimportant compared with the loss of an abutment.

11.35. Length of guide-bank. There is no definite rule about guide bank length. Regime theory indicates that any rule can be framed logically in terms of meander length and, therefore, of flood breadth because of equation (10.22). The author prefers from about three-quarters to one flood breadth, sees no advantage
in banks exactly at right angles to the bridge or even completely straight ones, and believes in locating, if expedient, to take advantage of physical features that reduce cost. Because the banks do not fully straighten the flow (particularly in rivers that can deposit shoals during years of low flow) although they do improve approach conditions to the bridge, there is advantage in continuing them about one quarter flood breadth downstream of the bridge.

11.36. **Spurs upstream of bridges and barrages.** This text cannot discuss the many practical reasons that affect location of spurs to improve approach conditions to bridges and barrages, or even to substitute for guide banks. However the first spur upstream is normally expected to confine the meandering tendency of the river, so it ought to be located to break the half wave length; this places it at about 0.4ML upstream, as recommended by Inglis, 1949. Equation (10.22) makes this distance about four flood breadths; however, the actual meander length of the river is the best measure unless braiding makes it indeterminate.

11.37. **Rules for estimating scour.** Any literal obstruction—pier, abutment, guide bank, etc.—causes a concentration of flow in its vicinity and, therefore, a scour hole. So a standard problem of any training work is to estimate what depth of scour hole might occur and then take steps to counteract or prevent it. Unfortunately there is no good theoretical way to estimate scour. A useful practical way, based on Indian experience, is to visualize any particular type of obstruction as causing a fractional enhancement of ordinary straight channel regime depth. This idea seems to have arisen through an opinion of Lacey, 1929-30, to the effect that, as the mean velocity in a curved reach of a regime canal is about the same as in a straight one, and as the worst sectional form imaginable is a right-angled triangle, therefore the worst total scoured depth at a bend should be twice the average. Inglis, 1949, published a list of extreme scours round bridge piers in sand rivers in India, plotted the results, and found that, on an average, between 30,000 and 3,000,000 cusecs, they represented almost exactly twice the (canalized) regime depth he estimated for high flood.

11.38. **Flood depth and zero flood depth.** To use the preceding idea, despite its lack of theoretical justification, because it does provide a means of codifying experience, one must define a “flood depth” to correspond with the “flood breadth” of para 11.32. Then a special problem arises about rivers with large bed-load charge.
The scour around, say, a spur nose is due to quite a different flow pattern than causes the scour at the outside of a smooth natural meander-bend; for oblique flow it seems due to a kind of whirlpool action. Therefore the depth of hole seems likely to depend on bed-material size and not at all on bed-load charge until the charge becomes relatively large. Accordingly the author recommends using a **zero flood depth**, \(d_{fo}\), as a basis for scour estimates. This may be defined as the regime depth of a canal in the proper phase, having the zero bed factor and discharge of the river, and discharging through a breadth equal to that estimated for the flow channel of the attacking stream (or the flood breadth) diminished by the obstruction.

11.39. **Scour depths for design.** As a practical guide for estimating the scoured depths to use in designing aprons (para 11.41), but not to obtain the scours that might actually occur with or without them, \(d_e\) or \(d_{fo}\) may be multiplied by factors \(z\) as follows:

- Severest attack on natural meander bend
  \[ z = 1.7 \text{ times } d_e \]
- Abrupt impingement of flow on long bank
  \[ z = 2.0-2.25 \text{ times } d_e \]
- Between and around bridge piers
  \[ z = 1.5-2.0 \text{ times } d_{fo} \]
- Downstream of barrages with hydraulic jump on floor
  \[ z = 1.75-2.25 \text{ times } d_{fo} \]
- Noses of spurs or guide banks or equivalent
  \[ z = 2.0-2.75 \text{ times } d_{fo} \]

These figures are based on practical conditions and include cases of very severe attack; they do not absolve the designer from using his own experience and judgement. They are in no sense theoretical.

11.40. **Model information on bridge pier scour.** Model work on bridge piers scour has received attention throughout the world, particularly in Europe (Laboratoire Nationale d'Hydraulique, France, 1956) and has been summarized by Neill, 1964(b). The mechanism of the flow in the scour hole is now clear. A satisfactory design formula has not yet emerged, but Neill finds that model results to date, for the deepest unrestrained scour that can occur at any velocity for a given approach depth, are fitted fairly well by the equivalents:

\[
\frac{d_s'}{w} = 1.5\left(\frac{d}{w}\right)^{0.3} \quad \text{...........................................................11.10)}
\]
\[
\frac{d_s'}{d} = 1.5\left(\frac{w}{d}\right)^{0.7} \quad \text{...........................................................(11.10a)}
\]
where \( d \) is the depth of local approach flow, \( d_s' \) is depth of scour below bed of local approach channel and \( w \) is pier breadth projected at right angles to the flow; for impact on the nose shape is assumed rectangular—for circular noses 1.5 should be replaced by 1.2; for oblique impact, when \( w \) becomes projected breadth, the 1.5 remains regardless of nose shape. The size of bed-material does not enter, although experiments show that coarser material than sand can give slightly deeper holes. A. S. Qureshi, 1965, finds that lightweight materials give shallower maximum depth of holes. The deepest scour is said to occur when the bed is somewhere round the dune-forming state. The formula

\[
d_s/d = 1.8 (w/d)^{1/4}
\]

(11.11)

where \( d_s \) means depth from water-surface to bottom of scour, which fits certain model experiments (with small \( C \)) of Inglis gives practically the same answers as equation (11.10) until \( w \) exceeds \( d \); it suffers the theoretical objection that it predicts no flow depth if \( w = 0 \). It is, practically, a useful formula for problems of likely scour under specified circumstances e.g. round a cofferdam at a given discharge; the \( z \) factor, 2.0 of para 11.39 merely gives an estimate of the worst ever likely to happen under any practical circumstances including curved flow and oblique approach so is applied to \( d_{so} \), not to \( d \). Although a better theoretical approach to bridge pier scour is developing the facts remain that there may be phase differences between models and prototypes, and field observations of flood scour are negligible. Therefore equations (11.10) should be applied to practical cases with caution. Para 2.16 details the constituents of total scoured depth.

11.41. **Flexible aprons.** Banks for any kind of river training work need protection right to the bottom of whatever scour hole will form. Only flexible protection such as stone, concrete blocks, or, sometimes, articulated concrete matting, is likely to have any permanence on a yielding base. Normal procedure is to build a bank only above a convenient level, lay the protection on its side slope and then lay the protection that will be required for the rest of the bank (after scour has occurred) as an apron (Fig. 11.2) on the natural ground. Provided the ground is non-cohesive, the stone will settle to a slope of about 1 upon 2, when it is undercut by scour, and will thus complete the protected bank. The angle to which the stone will settle depends on the properties of stone rolling on stone and not entirely on those of the non-cohesive
material underneath; hence the fair accuracy of the slope 1 upon 2. If the ground is cohesive the apron is likely to remain on a cliff as scour occurs; when the cliff fails by sliding, it will carry the apron end away where it will be useless and, eventually, the bank will be lost. Practice has normally been to lay the apron over a length of 1.5H, where H is the depth of estimated scour below the position of laying; the scour d, is estimated from empirical rules such as in paras 11.39, 40. Model experiments by Inglis, 1949, show the mechanism of apron launching and demonstrate that the length of laying can be greatly curtailed provided that the proper volume of stone is retained. The volume for a two-dimensional problem is obviously \((5)^{1/2} HT\) where T is the thickness needed on the finally settled slope.

11.42. Apron stone size and thickness. The size of stone to use depends considerably on the conditions of laying. If an apron is laid at a high level it may come under relatively sudden violent attack at some time and be carried away before sufficient scour develops for it to settle; demonstration of this occurrence round bridge piers has been made on models by Inglis, 1949. There seems no theoretical way to devise a formula for a stone size. For stone on a canal bed or bank the Meyer-Peter, 1948, formula, or equation (14.12), or tractive force principles, Lane, 1955, suffice. In rivers where oblique attack occurs, flows may be supercritical, the phase may be that of small \(d/D\), and stone may be laid at high level and attacked suddenly, experience seems the best guide. Individual designers can make alternative calculation of their own as an aid, and may expect considerably different answers from different methods. A rough guide is that a large sand bed river will normally need stone about 150 lb. if it does not have a very large bed-load; a small one might have stone as small as 50 lbs. A gravel river with small bed-load charge should use stone at least twice the diameter of the largest material that rolls on the bed, if moderate attack is expected; for very violent attack, as at a major spur nose, three times size is safer. Some gravel rivers have very large bed-load charge, so the size of stone on the bed is not an indication of the force of attack. Fig. 11.3 (California Dept. of Public Works, 1960) gives recommended stone sizes for different flow velocities and different types of attack; the ruling for different attacks is that the mean velocity of flow should be enhanced by one third to allow for very violent attack, and reduced by one third for situations where there is practically no attack. The basic formula
is open to theoretical dispute but the chart does give stone sizes that accord with experience and does make an allowance for different specific gravities. The reference gives good practical advice on stone specification. Fig. 11.4 shows some alternative design criteria from reputable sources. Fig. 10.4 suggests that D should depend on V,d and not on V alone, but practical conditions are too indeterminate to justify refinement of present practical methods.

11.43. Stone thickness cannot be less than one layer, nor is a fraction of a layer practical. Therefore normal specification for bank protection is stone equivalent to two layers. In the apron the amount is that to fill the finally launched bank two layers thick. As stone tends to launch in a single layer there is a fair reserve in an apron and this compensates for the necessarily rough z values in para 11.39. If stone is to be placed on a fine-grained soil bank a filter layer of gravel should be given as an underlay to prevent wave-wash and swirl from pulling soil through the stone interstices.
CHAPTER 12

RIVER MODELS

12.1. Introduction. A river model, to be effective, must be just a small river, so an experienced river engineer is in a position to assess its meaning. It has the advantages that the discharge can be regulated at will and the bed can be inspected in detail anywhere. Like almost every model of any kind, be it of an earth dam, bridge, building, boat, or rigid-boundary hydraulic situation, it is liable to exaggerate the relative importance of factors that are relatively unimportant in the prototype—as we shall call the full-size set-up. It has a host of other disadvantages, including that some of the factors it would like to represent cannot be measured in the prototype so cannot be copied, others cannot be reproduced by any economic means, and some cannot be represented at all. Thus, the really expert model-maker should understand river engineering, mobile and rigid-boundary hydraulics, some other branches of physics moderately, and the peculiar distortions produced by models; in addition he must have remarkable ingenuity and patience. As few men are gifted enough to acquire all these qualities (or likely to receive the facilities for acquiring them even if they were), model work should be co-operative, with the practising river engineer taking a fair share in deciding whether a model, with expert assistance, is necessary, what he may expect from it, and what it means when completed and running. The present chapter uses basic principles and formulas to assist the river engineer’s judgment when he takes this share; it is not intended specifically to instruct model-makers.

12.2. Small canal as model of large one. To appreciate the simplicity and fundamental importance of calculating model scales let us suppose that an engineer, knowing only rigid-boundary hydraulics, was told to make a 10 cusec model of a 10,000 cusec regime canal of the type of Chapter 5; to make his task particularly easy he was permitted to make it alongside the canal, in the same soil, using water-sediment complex drawn representatively from it. In technical jargon he would have been told “make a model to 1/1,000 discharge scale”. He would almost certainly design his model geometrically similar to the prototype (canal) and at the same slope, with the dimensions calculated from Manning’s formula assuming
the same n. As we know from Chapter 8, the model would be far too flat to carry its load, so bed load would deposit in the head reach and the model downstream would not have dunes of sand on its bed and, therefore, would be uncharacteristic of a regime channel; if he persisted in running it long enough bed material would pile up and increase the slope, while shape would alter, till eventually he would have a channel with $1000^{1/6}$ times the slope and depth to breadth ratio with which he had started, and it would then run in regime like the prototype.

12.3. **General concept of a model.** The result would probably puzzle him for, in rigid-boundary hydraulics, “dynamical similarity” cannot exist without geometric similarity; yet here is an example where a model will move its bed material just as in the prototype if there is great geometric dissimilarity, and geometric similarity does not produce the principal phenomenon of the prototype. Actually the bed-material size ought to be shrunk in the same ratio as depth to achieve full geometric similarity, but then the model bed-material would probably be cohesive. Anyway conventional dynamical similarity would still require $V^2/gd$, i.e. $F_b$, to be kept unaltered, and this forbids shrinking the bed-material size. Without entering into theory it is clear that mobile-boundary work is likely to require a more general type of model than the dynamically similar. In general a model may be defined as “something having features or happenings that can be placed in correspondence with those of a prototype”. For quantitative predictions the correspondence is calculable.

12.4. **Examples of models.** A Mercator’s projection, with contours, is a model of the earth; the correspondence is provided by calculable mathematical laws that are used by navigators. The electric analogue of hydraulics uses a sheet of graphited paper and a Wheatstone bridge set-up to model hydraulic flow through the pervious soil under a barrage, with a correspondence provided by the fact that the laws of electric flow and subsoil flow following D’Arcy’s law are mathematically identical; as calculation is difficult the model performs it. A small sand-bed canal is a model of a large one, with calculable correspondence provided by the formulas of regime theory (Chapter 7).

12.5. **Rivers as models of each other.** Fig. 3.2 shows how well certain river dimensions can be put in correspondence with each other. Every river used in the plot of a river system line in that Figure is put into correspondence with every other river so far as $b$, $d$, $V$, and $Q$ are concerned; the equation of a line, of course, will serve
as well as the line itself. The plotted points of each individual river of a system do not fall exactly on its system line, because there are factors at work besides the principal ones plotted, but this merely means that perfectly calculable correspondence within a system is not possible. Every system represented by a line is a model, through any of its members, of every other system with a parallel line because of the obvious correspondence, which can be reduced to a formula such as

\[ b = BQ^{1/2} \]

; this provides calculable correspondence when the appropriate B is known. This knowledge may come from the fact that B must be almost exactly \((F_b/F_s)^{1/2}\) for a system, or just by measuring it off the graph. Finally every system, whatever the slope of its characteristic line, is a model of the others in the sense that there is correspondence that can be picked off (Fig. 3.2), but practical calculability has vanished.

12.6. Calculation of model scales. A model scale for some quantity—say discharge—means the ratio of the model quantity to the corresponding prototype one. The word “model” always implies a comparison with something else which is then called the prototype; either partner may be called the model. If one of the rivers has been constructed in a laboratory it is normally called “the model”. The term “corresponding” is used only in comparing the same physical quantities in model and prototype—e.g. discharge with discharge, breadth with breadth—so may be taken as meaning that the comparison is between the same fractions of the mean value in model and prototype. The calculation of a scale requires acceptance of an equation that is valid for both model and prototype and the algebraic procedure is to insert the model dimensions in the equation, then insert the prototype dimensions and divide the first equation by the second. Thus, if equation (7.8) is valid, and suffices m, p refer to model and prototype, the scale equation is:

\[ \frac{b_m}{b_p} = \left[ \frac{F_{bm}}{F_{bp}} \right]^{1/2} \left[ \frac{F_{sp}}{F_{sm}} \right]^{1/2} \left[ \frac{Q_m}{Q_p} \right]^{1/2} \]

which is written much more simply as:

\[ b = (F_b Q/F_s)^{1/2} \]

where R is agreed to mean that every term in the equation is a scale. So, allowing for the fact that dividing an equation by itself, after inserting m, p quantities, cancels out any constant coefficient,
the scale equation is just the original from which it came, with any constant coefficient removed; the same artifice of devising an (R) equation was used in para 11.24. If radically different equations apply to model and prototype a simple equation like (12.1) cannot be found and scales will be variable. In most of this Chapter problems assume, for the sake of illustrating method, that the difference is small enough for the regime theory phase equations to apply to both model and prototype.

**Problem 12.1.** A model-maker has to model a tidal river whose peak flood from above the tidal reach is 1,000,000 cusecs. He has to accept a horizontal scale of 1/1,000 to suit the plot of land available. Like everybody else he sees no reason to make the scale different in different directions; automatically this makes his meander slope correction coefficient (k of equation (11.1)) the same in model and prototype. Because tidal currents are important he decides that he will manipulate the model till he gets the Froude Number in terms of depth (and therefore $F_b$) the same in model and prototype—i.e. the $F_b$ scale is to be 1.0. To achieve this he may have to use sand finer than the river’s (Fig. 10.4) or light-weight material. Because he is not sure whether an exaggerated side-factor would result in a wrong phase of flow at the sides, he decides that he would like to keep it the same as in the prototype—i.e. $F_s = 1.0$ . . . (R). Because he is satisfied, from river analysis, that the bed-load charge of the river is small, and he cannot measure it anyway, he reckons that he can experiment in the model with charges round the estimated small amount and, in fact, use manipulated charge to get the model running satisfactorily—virtually he aims at $C = 1.0$ . . . (R). He assumes that viscosity in the model will be near enough to that of the river to make no difference—virtually $\nu = 1.0$ . . . (R). Because he knows the regime equations he knows that he cannot make any more decisions. (To do so would be to assign more conditions than the equations have independent variables and, therefore, ask the model to perform an impossibility). So his immediate practical problem, for building the model, is to find the depth, slope and time scales that the model will choose for itself, because it is a river subject to the regime laws, to be consistent with the preceding decisions. (Obviously, if he builds the model to these scales, he will not have to waste effort running it to various impossible conditions till trial-and-error shows how it wants to run). What are these scales? Specifically, hav-
ing found them, what time should be allowed on the tide-generating machine for a 23-hour tide-cycle, and what time would be required to run through one year's upstream river hydrograph? What discharge represents 1,000,000 cusecs?

**Answer.** Equations (7.8, 9, 10'), written in ratio form, reduce to \( b = Q' \), \( d = Q' \), \( S = Q' \). So \( Q = b^2 = 10^6; d = 10^{-2}; S = 10 \ldots (R) \). The time scale \( T \) is the ratio of the times of corresponding events; the time for a particle to move a distance \( b \) with the mean flow velocity \( Q/bd \) may be taken as the event, making \( T = b^2d/Q = 10^{-2} \ldots (R) \). So a tide-cycle on the model should be 0.23 hours, and the time to run through a year's hydrograph is 3.65 days. The high flood in the model would be 1 cusec. The answers are first order approximations since the equations used to obtain them are for ideal conditions; the model-maker should be expected to adjust them slightly while "proving" the model (para 12.7).

12.7. **Importance of initial scale calculation.** A major model may run for years before useful results are obtained, and a considerable portion of the time may go in the routine of "proving". "Proving" consists of trying to make it reproduce some river history relevant to the task in hand; if reproduction is poor then various adjustments, including scale changes, are made till it is good. If the initial scales are inconsistent with regime equations (applicable to the phase) then a model cannot reproduce the river regime and proving will consist, virtually, of trying to discover the regime laws by trial and error; the time required for this may be great.

12.8. **Analysis of model scales.** In Chapter 11 analyses of river data were made by regime formulas to test the applicability of the formulas. Comparable action on the rivers that are models is difficult because the various parameters required for the purpose are not readily and reliably obtainable from reports. However, final breadth and depth scales are usually on record and Table 12.1 given them for a set of models of some prominence that seem to have been mostly of regime type and are alleged to have produced useful engineering results. The reciprocals of the scales are plotted against each other in Fig. 12.1 (from Blench, 1951a) to test fit to the relation:

\[
d = \left( \frac{F_s^{2/3}}{F_0} \right) \cdot b^{2/3} \quad (12.2R)
\]

which is found by simultaneous solution of equations (7.8, 9). A band is described about a central line of slope 2/3 to cover values
Table 12.1. Actual width and depth scales

<table>
<thead>
<tr>
<th>Point</th>
<th>Tidal or not</th>
<th>Location of prototype</th>
<th>b</th>
<th>d</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>T</td>
<td>Zeebrugge</td>
<td>400</td>
<td>60</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>T</td>
<td>Abidjan</td>
<td>270</td>
<td>120</td>
<td>See Donovan</td>
</tr>
<tr>
<td>3</td>
<td>T</td>
<td>Hook of Holland</td>
<td>1000</td>
<td>150</td>
<td>Lee in &quot;Dock &amp; Harbour&quot;</td>
</tr>
<tr>
<td>4</td>
<td>T</td>
<td>Wilhelmshaven</td>
<td>200</td>
<td>75</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>T</td>
<td>Severn</td>
<td>8500</td>
<td>100</td>
<td>Authority&quot;</td>
</tr>
<tr>
<td>6</td>
<td>T</td>
<td>&quot;</td>
<td>8500</td>
<td>200</td>
<td>June, 1949 for original data</td>
</tr>
<tr>
<td>7</td>
<td>T</td>
<td>Rangoon</td>
<td>8060</td>
<td>192</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>T</td>
<td>Parrett</td>
<td>3000</td>
<td>260</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>T</td>
<td>Cheshire Dee</td>
<td>5000</td>
<td>200</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>T</td>
<td>&quot;</td>
<td>40000</td>
<td>400</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>T</td>
<td>Great Ouse</td>
<td>2500</td>
<td>60</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>T</td>
<td>Humber</td>
<td>7040</td>
<td>192</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>T</td>
<td>Mersey</td>
<td>7040</td>
<td>190</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>T</td>
<td>Seine</td>
<td>40000</td>
<td>400</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>T</td>
<td>Penang, Malaya</td>
<td>1500</td>
<td>75</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>T</td>
<td>Prai, Malaya</td>
<td>600</td>
<td>150</td>
<td></td>
</tr>
</tbody>
</table>

I            N            Mississippi    600 | 120 | 110-1/1938
II           N            Mississippi     600 | 100 | 159-1/1940
III          T            Gavelston Bay, Tex. 800 | 80  | 127-1/1940
IV           N            Ohio River      300 | 80  | 181-1/1941
V(a)         T            Delaware River 800 | 80  | 194-1/1942
V(b)         T            "             "     "   "     2-231/1947
V(c)         T            "             "     "     "     2-259/1948
VI           T            Abescon Inlet, N.J. 500 | 100 | 204-1/1943
VII          T            St. John’s River 1000| 100 | 2-244/1947

of the coefficient from 1/2 to 2; most of the points that fall outside it are for tiny models of such enormous vertical exaggeration that one cannot imagine them being used today. All the American large models are in the band and some right on the centre line. The generally good fit to a relatively tight band, like the good fit of many rivers to the Regime Slope Analysis Chart of Figure 3.3, does not prove that regime-theory formulas apply to all phases. It is
consistent with most rivers of engineering interest running in that phase, and with model-makers being able to manipulate models by trial and error (after more or less time) to suit. The reasons for this ability are simple. The rivers had sand beds and normal bed loads. The tidal models must be given unit Froude Number scale, which can be achieved by using fine or light-weight material; similar materials ensure the proper phase in non-tidal models where the restriction on the Froude Number is not mandatory. Slope scale can be left to adjust itself without much embarrassment—whence, probably, the lack of records about it. Time scale for rates of erosion and transport is a major obstacle avoided by refraining from prophecies about these quantities.

12.9. The generally good fit to a relatively tight band, despite the modeller’s ignorance of phase changes and the specific mobile-bed formulas for steady canal conditions, raises various questions. What methods have been used to find scales? How much of model-making is art and how much science? How exhaustive are the quantitative predictions of even a large sophisticated formal model of the complex conditions of a river? Could the long proving time of some models be reduced if more basic knowledge were available? Clear answers would be valuable to the administrative engineer deciding whether to commission a large model, a small one, or none at all. Simple answers are not available, but an assurance can be given that every kind of model can be economic under the proper circumstances. Therefore, the following paragraphs will attempt a semi-technical informal discussion and illustration of scaling and model limitations; the intention is to draw attention to matters that deserve consideration before an administrative decision.

12.10. **Dynamical similarity.** The quantitative model-maker’s dream is that he can impose certain constant scales on a model (Problem 12.1 demonstrates that arbitrary imposition is limited), calculate the remaining ones that the model will then impose on itself even if he does not arrange to anticipate them, measure model quantities, and then use the scales to prophesy the prototype ones. In theory the “dynamically similar” model realizes the dream and calls for no knowledge of hydraulic formulas. But it requires him to (i) state the prototype case (as in para 10.12), (ii) put it into dimensionless parameters (numerics) as in Eqs. 10.2, (iii) impose any three dimensional scales involving all of mass, length and time and (iv) impose the condition that every independent numeric must have unit scale (that is, have the same value in model and prototype).
When this is done every dependent numeric scale is unity and a glance at Eqs. 10.2 shows that every dimensional scale follows from the three initial arbitrarily imposed ones. Unfortunately the only practical fluid for a model is water, so the \( \rho \) and (very closely) the \( \nu \) scales are fixed at 1.0; and our location on earth fixes the \( g \) scale at 1.0. Therefore, all dimensional scales are 1.0, which means that the only dynamically similar model of a river is itself. (The general theorem is given by Yalin, 1965 and Blench, 1968a.)

12.11. Practical scaling. To illustrate practical scaling from ideas of dynamical similarity, aided only by rigid-boundary formulas, suppose the following prototype data for Problem 12.1. Length of interest 150,000 feet; maximum flood depth in the navigation channel at the mouth 50 feet; corresponding breadth of stream surface 1,500 feet; maximum tidal swing 15 feet; bed material size 0.25 mm. The 1/1,000 length and breadth scale depends on available model space, and not on theory. Assume the modeller had none of the special knowledge of this text but knew conventional rigid boundary hydraulics and its elementary dimensional analysis that does not go far enough to prove that dynamical similarity is not strictly possible here. He would like geometric similarity if only because it is essential for dynamical similarity. However, this would give 0.05 feet maximum model depth and probably about 0.02 feet in fairly important channels upstream. These are too small for accurate measurement, would obviously lead to a Reynolds Number near to laminar flow and might introduce complications from surface tension. We can imagine he decided on a depth scale of 1/100 to get six inches maximum depth of navigation channel. The navigation channel will then have 1.5 feet surface breadth and 0.5 feet depth, with a b/d ratio of 3.0, discussed in para 12.12. The next step would be to prescribe \( V^2/gd = 1.0 \ldots \) (R) which is mandatory for tidal work and common in rigid-boundary models without tides; it is necessary in dynamical similarity, but not sufficient. Guessing tide speed at 15 ft/sec maximum the model speed would be 1.5 ft/sec, giving a model discharge of a couple of cusecs, which can be accepted as easy to handle. The discharge scale would be, then, 1/1,000,000. A bed-material scale of 1/100 would preserve geometric similarity in longitudinal section, but would call for material of 0.0025 mm. size, which is colloidal and neither noncohesive nor bed-material. Obviously sand has to be used in the model and, unless the model were remote from the prototype, the obvious sand is that from the river bed. (Thus the major scales of the original Problem 12.1 have
arisen from force of circumstances, happen to fit regime theory equations, and disagree with dynamical similarity principles at which they started to aim). For model slope Manning’s formula with a guessed n would probably be used in the knowledge that bed-material would have to be injected by trial and error anyway and might give the desired slope; if it did not the model would be allowed to adjust to the slope it preferred.

12.12. A conspicuous defect of this model (assuming for the present that it would run in the prototype phase) is the b/d ratio of 3 at high flow. It would call for stabilizing (probably in plaster) the river and island shores at about 3 upon 1 slope to represent their probable natural 1 upon 3 if they were sandy. Scour holes in sand around obstacles would be expected to assume the 10-times exaggerated slopes also and, of course, would refuse to comply. So the model would not prophesy happenings along the shore—say at wharves for local industry—due to actions such as major training works to improve the navigation channel, and would be of doubtful quantitative value for erosion predictions.

12.13. The value of d/D for the model would be about 600 in the deep navigation channel and possibly of the order of 300 in the main channel upstream; at low tide in the navigation channel it would be about 400. Fig. 10.4 suggests, therefore, that the model channel would not form dunes ordinarily; it might even refuse to move its bed. Had bed material been of 0.45 mm size movement would have been still more unlikely. This is the trouble that has forced model-makers into the common-sense action of using smaller bed-material than the prototype and, if that failed, light-weight material. The last works well and is now common, but calculated scaling has remained difficult because of lack of reliable transport formulas and inability to measure prototype bed-load. An attempt to scale bed-load charge brings to notice that all scaling is for average-steady conditions, since available formulas concern only steady flow; the hope is that the same fixed scales will look after fluctuations about mean conditions also (paras 9.2-4). Now, if cognizance is taken of C at high flow in Problem 12.1, then $F_b = 1.0 \ldots (R)$ implies $F_{bo}(1+0.12C) = 1.0 \ldots (R)$, which makes $C_m/C_p$ depend on $C_p$, and therefore, on stage, unless $F_{b o} = 1.0$, in which case $C=1.0$. This variation of charge scale with stage does not matter unless problems concern rate of erosion or deposition, when it affects accuracy of prediction. Until good formulas are available for steady bed-load transport such a complication is rather aca-
Some general limitations on models are the impossibility of reproducing rates of erosion of shores or banks that are cohesive; great difficulty in reproducing the natural variation of bed-material size along channels and between channels; greater difficulty in reproducing total load and, therefore, the shoaling it causes; and the impossibility of reproducing depositions due to density currents in estuaries without using two fluids. Two-fluid models are specialized and have recorded successes (Inglis & Allen, 1957; Inglis & Kestner, 1958; Price & Kendrick, 1962; Schultz & Simmons, 1957).

**The Small-Scale Large Model.** The type of model just used for illustration could be commissioned by a major organization responsible for continuing river-works costing, perhaps, millions of dollars annually. Then several hundred thousands of dollars of model work over a few years would appear as a moderate insurance of the reliability of technical advice on the means for reducing costs. The first model, covering a great length and having an unavoidably small scale, is regarded as a pilot representing general river regime and indicating the conditions to apply to subsequent large-scale models of local situations (Inglis, 1949). The potential for financing concurrent international basic field and laboratory research is great and, if exercised, would permit institutional researchers to expand investigations to field scale and to conduct them in association with practising engineers.

**Basic Quantitative Models.** The antithesis of large utility oriented model is the tiny laboratory one that aims at discovering only mechanisms and laws of behaviour. Suitable topics include density currents, sediment waves consequent on sudden changes of fluid flow, the relation of apparent viscosity to concentration of solids, and the direct measurement of shear stress on rough boundaries, Rajaratnam, 1968. A recent example of a basic model that may remedy present dubious scaling of bed-load charge and rate of erosion is by Yalin, 1965. Virtually he modelled the prototype canal case of para 10.12, using light polystyrene to represent quartzose sand and tested the hypotheses that those equations could be reduced by (i) removing s, (ii) replacing g by g(s−1) except where gS occurred, (iii) replacing C by C/s. (Actually he used different numerics, but the case is stated here to suit the numerics of this text). The beliefs behind the hypotheses were that, in the real
unknown) equations for average steady flow, the only way in which the g of Eqs. 10.1 could appear is in association with buoyant weight of sediment, and that charge would appear as volumetric charge instead of charge by weight. They call for reducing Eqs. 10.2 to:

\[
\frac{V^2}{gd}(s-1), \frac{V^2}{gdS}, \frac{\lambda}{D}, \frac{\alpha}{D} = \text{fns} \left( \sqrt{\frac{V}{g}}(s-1) \frac{D}{v}, \frac{C}{s}, \frac{d}{D}, \frac{b}{d}, X \ldots \right) \quad \ldots \quad (12.3a, b, d, e)
\]

The test of the hypotheses was to model the bed-waves on a sand bed in one flume by polystyrene in another, using the scales \( g = 1.0 = v \ldots \) (R) and the standard dynamical similarity conditions that every numeric on the right should equal 1.0 . . . (R). The conditions give \( b = d = D = (s-1)^{-1/3} \) and \( C = s \ldots \) (R). Then the left side gives \( V = (s-1)^{1/3}, Vd = 1.0 \) and \( S = (s-1) \ldots \) (R). The waves came out similar and the derived scales were verified very closely (indicating that differences in \( X \ldots \) were unimportant).

12.17. Obviously polystyrene models scaled as above would not be practical since they would be larger than the prototype and have the same discharge intensity. (The \( s \) values for polystyrene and sand were 1.035 and 2.65, so the length scale had to be 3.6 and the slope scale 1/47). However, one implication from the results is that the regime-theory equations could be converted to use for lightweight materials by replacing \( \frac{V^2}{gd}, \sqrt{\frac{V}{g}}D/v, \text{etc.} \) by \( 1.65 \frac{V^2}{gd} (s-1), 1.65^{1/3} \text{times} \sqrt{\frac{V}{g}}(s-1)D/v, \text{etc.} \) The coordinates in Figs. 10.8, 9 could be treated similarly with \( S \) becoming \( 1.65S/(s-1) \). The numerical multipliers merely arrange that the expanded quantities collapse to the ordinary ones when \( s = 2.65 \) and \( v = 10^{-5} \).

12.18. Value of Qualitative Models. The theoretical and practical difficulties that lie in the way of copying river situations to definite scales, and consequently in the way of making quantitative predictions from models, do not prevent the more qualitative models from having great utility to an experienced river engineer. The reproduction of almost any type of obvious river situation, that can be studied fairly within the length of a couple of meander bands, requires very simple facilities, and can often be done best in the field where a copious and cheap water supply will avoid the distortions of a tiny laboratory river. Some trial and error will get a model river running in a manner recognizably comparable with the prototype’s, and the trial and error will be slight if regime formulas for the proper phase are used to assist. Then questions
illustrated by the following can be answered with practical engineering utility. Will vanes in front of a canal regulator be efficient in diverting bed-load away from the canal? Will a proposed groyne at a barrage produce disturbances likely to affect sediment entry into a canal? Is a proposed pump-house location on a river likely to result in minimum entry of bed load into the intake? Will a bridge without guide banks be outflanked by meandering, and what severity of erosion must be expected along the approaches and at the piers and abutments if it is? Will a jetty built out to and up an island attract erosive attack or deflect it to the opposite shore? Can a persistent shoal be reduced by spurs or dikes? Where are the aprons of a submarine crossing likely to receive most attack? How can a safe river diversion be made with the aid of stone dikes? When the engineer has the answers he can use methods like those of Chapter 11 to assist in estimating the magnitudes of the effects that the model scales might predict wrongly.

12.19. Flume models are useful for studying phenomena in two dimensions; in a way they represent a slice through a river. Popular subjects for study are scour below drops, the size of stone for rock rapids and river diversion dikes, and the settlement of aprons along dikes and weir floors and around bridge piers. Because some of these problems are rigid boundary ones that do not suffer much from scaling complications good quantitative accuracy is often possible.

12.20. Perhaps the models that have benefited engineering most are those devised to study aspects of river behaviour qualitatively. Typical are studies of how spilling rivers meander (Friedkin, 1945), how scour develops around piers (Tison, 1940, Inglis 1949; Neill, 1964 (b)), how flexible aprons launch (Inglis, 1949), how bed-load moves (Gilbert, 1914; Simons & Richardson, 1966). Such studies deserve additions and parallels in the field; neither the laboratory nor the field is satisfactory alone. Regime calculations, as in Problem 12.1, are usually good enough to permit the major dimensions of a qualitative model to be estimated so that it can be put into operation rapidly and they do not need to be very accurate. The following problem is worded to show how the set-up for the very realistic model of Plate 16 was planned.

12.21. **Example of Crude Scaling for Qualitative Model.** Imagine a braided river rather like Plate 14 but not so wild. A bridge with guide banks, as an innovation, is to be built to a tentative spanned breadth of 350 feet with guide banks about the same length; the
previous bridges had a history of damage during spates. The engineer asks, at short notice, for a model to demonstrate locations and relative intensities of attack on the training works, but prefers to use his own experience and formulas (similar to those of Chapter 11) to decide amount and sizes of riprap. Information given to the laboratory amounts to (i) air photos, (ii) local contour plan of river, (iii) design flow 40,000 cusecs, estimated median annual peak 8,000 cusecs, (iv) bed material in thalweg about 3 inches median size and finer on shoals, (v) slope locally straight down the flood plain 0.4%, (vi) height of high flood above estimated mean bed about 10 feet, and height of 8,000 cusecs above it 7 feet, (vii) one site visit by the modeller, (viii) history of damage and repair to old bridges. Here there is not a definite river channel to study and transport is not in the phase for which good factual formulas exist, but the model is small enough to be forced rapidly into fair representativeness; it can be forced faster with the aid of crude calculations. One of various lines of argument could be as follows:

i. Decide $b = 1/100$ . . . (R) to fit 2,500 feet of river into available lab space. Accept $d = 1/50$ . . . (R) to give a moderate depth and little vertical exaggeration.

ii. Design for 8,000 cusecs because of its prevalence and small spill. Checking for phase, $(d/D)_p = 84/3 = 28$, so Fig. 10.4 shows $F_{bo}$ in Kellerhals phase. Accept 0.25 mm sand for model as it is available; then $(d/D)_m = (84 \times 25.4/50)/0.25 = 170$ and the model is well outside the regime-theory phase.

iii. Therefore, $F_{bo} = (D/d)^{1/2} = (28/170)^{1/2} = 1/2.46$ . . . (R), using the centre-line of Fig. 10.4.

iv. To make model banks yield like the prototype ones, in braiding, take its “working side factor” and the prototype’s on the same basis as if for non-braiding. That is $F_{sm} = 0.15$, say and, $F_{sp} = 3.0$ from Eq. 10.21 using regime-theory $F_{bo}$. Then $(V^3/b) = 1/20$ . . . (R) and $V^3 = 1/2,000$ . . . (R), $V = 1/12.6$ . . . (R), $Q = bdV = 1/63,000$ . . . (R); $F_b = V^2/d = 50/159 = 1/3.2$ . . . (R). Note that $Q_m$ for 80,000 cusecs is a little over 1 cusec, so manageable.

v. To estimate the C scale note that $F_b = F_{bo}(1+0.12C)$ . . . (R) so that $2.46/3.2 = (1+0.12C_m)/(1+0.12C_p)$.

vi. To estimate $C_p$ use the Z diagram, Fig. 3.3. For this assume $b = 2 \times 8,000^{1/2} = 180; b^{1/6} = 2.38$. Take S about 2/3 of slope down the valley = 0.27%. Then $Z = b^{1/6} S/F_{bo}^{11/12} = 2.38 \times 0.27/6.1% = 1.05$ per thousand. Plotting Z against $Q = 8,000$ indicates C
about 10 for \( k = 2.0 \); at 40,000 cusecs C scales about 18 from Fig. 7.2.

vii. Using \( C_v = 10 \) in para v, \( C_m = 5.7 \). This gives a rough figure for model injection for running at 8,000 cusecs; it would be about 2 pound per hour and the 40,000 cusecs flood would pick up enough to enhance this about 9 times. (The model would be run for a long period at 8,000 cusecs to let it develop different attacks; major floods would be imposed for short periods).

viii. To find model slope use \( S = F_b^{5/6} F_a^{1/12}/Q^{1/6} \), from which \( 1+C/233 \) has been omitted as useless. The use of the formula assumes that the stage with charge might be approaching the phase where the regime slope formula applies, but with \( F_{bo} \) according to Fig. 10.4 inserted (para 10.26). The answer is \( S = 1.85 \ldots \) (R), so the slope straight down the tray should be \( 0.4 \times 1.85 = 0.74 \% \); with tray length about 25 feet the drop is 0.17 feet.

ix. Time scale \( T = b^2d/CQ = b/CV \ldots \) (R) for erosion rate; it is obtained from the time required for a charge \( C \), derived from bed erosion, to pick up a specified fraction of \( d \) from a bed area of \( b^2 \). So \( T = (10 \times 12.6)/(100 \times 5.7) = 1/4.5 \ldots \) (R) at 8,000 cusecs, and about 1/40 at 40,000 cusecs.

The model would not be expected to work exactly to such calculations, but a little adjustment ought to settle it down. In fact the data apply to the model of Plate 16 which was built to a rougher set of calculations and gave no trouble. For demonstration it was started with a nearly straight channel. The photo of the Plate was taken after 90 hours of running at 8,000 cusecs; a record flood, imposed later, cut off the bend in a few minutes, thereby removing attack from the guide bank nose and giving favourable flow conditions through the bridge. Stone apron experiments were added. A principal lesson is that model experiments are best run by an engineer with river experience and sufficient knowledge of theory to appreciate its origin and limitations; cooperation amongst people sharing such knowledge is an alternative.
CHAPTER 13

SEDIMENT EXCLUSION AND EJECTION

13.1. Introduction. In bed-load transport the coarsest material tends to merely roll along the bed, or to saltate very feebly. In a channel bend the water screws itself round, instead of going round like an auto with the roof always over the wheels. The reason is generally known, and is still discussed along the lines stated by Thompson, 1876. Briefly, the curved flow can occur only if there is a horizontal pressure greater on the outside of a curved path than on the inside; so the surface at the outside of the curve superelevates to be consistent with the necessary pressure gradient. All down any one vertical the pressure gradient acting towards the centre of curvature has to be the same, since the cross-slope of the water-surface at the top of the vertical determines it. Therefore $v^2/r$ has to be the same all down the vertical, where $r$ is radius of curvature of flow. Therefore, as $v$ is smaller near the bottom, the bottom filaments have to move in curves of smaller $r$ than the top ones; that is, there is a secondary cross-flow, at the bed, directed towards the centre of curvature of the channel, and this will tend to move bed-sediment towards that centre; for continuity there must be an opposite cross-flow at the surface, tending to push floating material to the outside of the bend. Mockmore, 1944, demonstrates the effect in laboratory flumes. Within a channel the cross-drift of coarse bed-material with consequent size differentiation is important in canal and river work (paras 8.7, 9) but is not susceptible to calculation and is demonstrated best from canal experience. Therefore this chapter will be devoted to some descriptions of sediment manipulating devices that depend on curvature of flow and, to complete an instructive picture, descriptions of some that work differently.

13.2. Natural differentiation by curvature. Plate 17 (a) models a situation that existed in an old inundation canal system of India. The main canal (really a river creek) was for 7,120 cusecs, the right branch for 2,200 cusecs and the left branch for 4,920 cusecs. A natural equilibrium had established itself and the right branch supplied water to land of relatively low gradient. When the system was replaced by a modern one the main canal was
straightened and the right branch was taken off from a conventional head-regulator at right angles to the straight parent; it promptly aggraded and drowned out its head. Model investigation of the cause was by Inglis, 1949, who showed that moving bed-markers in the main canal moved to the left side of the approach bend and therefore failed to enter the right branch; coloured threads were used to demonstrate the screw motion imposed on the general flow. The best cure was obviously to make the approach curved once more; then the natural exclusion of coarse material from the right branch would lower its $F_{10}$ and restore its original regime slope. The natural sequel to discovery of the remarkable effectiveness of the curvature of the flow was to apply it to the entry to canals taking off from the right bank of the Lloyd Barrage, on the Indus. Plate 17 (b) shows Inglis' design, which was built and operated successfully. An S-shaped channel to the canal heads was built out from the right guide-bank (para 11.31) and connected into the third undersluice gate from the right of the barrage. Well along the outside of the first bend of this channel, where the coarse bed-material is certain to be absent, an approach channel is made to the canal heads.

13.3. The experiments for these works were accompanied by attention to the fact that various devices that exclude coarse material rely on the induction of favourable curvature of flow. One of the simplest is a plain divide wall built as in Fig. 13.1 (a). If the divide A cuts off just the right amount of flow, from the undisturbed parent to feed the offtake, the division of coarse bed-material will be fairly equable. If it is displaced outward to B then flow concentrates to the right of B so that the flow approaching B curves to the right, thereby inducing approaching coarse bed-material to move to the right bank and miss A. Conversely a wall built at C will curve the approach flow towards the left bank and concentrate bed-material there before it reaches C—then the offtake will become overladen with coarse bed-material. Other devices are given in the proceedings of I.A.H.R., 1951.

13.4. A device operating differently, and very successful with gravel, is a set of submerged vanes on the bed, opposite an offtake head, directed so as to deflect the bed-material away from the offtake, Fig. 13.1 (b). Plate 18 shows one in which several vanes were proposed originally, but modelling showed the best results with two; the case is special, in that the channel is curved. For sand beds vanes can work well at one discharge condition; for other conditions,
or if the vanes become obstructed, the flow is disturbed so that the sand is thrown into suspension.

13.5. Plate 19 shows tunnel excluders at the head of a major canal. One might regard them as vanes roofed at the elevation of the canal head regulator cill. The tunnels are controlled by an undersluice gate so that a good distribution of flows can be obtained at various flow levels. These devices can cause sufficient disturbance of flow in their vicinity to become relatively ineffective if the balance between river and canal flow is unfavourable, and if the bed-material is sand. The first of such excluders was designed and built without benefit of model tests and was given several tunnels. Later when the general utility had been established model tests were made and showed better results with less tunnels. A recent design is given by Karakie & Haynie, 1962.

13.6. Crump bed-ejector. The first device to eject sediment from the bed of a canal was designed and installed by E. S. Crump on the Upper Jhelum canal (9,000 cusecs) about 1933. This canal had its first 50 miles cut in foothills country and had not degraded to regime slope there; “level crossings” were used for a few very large cross drainages so the canals received an injection of relatively coarse sediment every time a crossing was operated. After some 30 years of running, the regime reach that ran in fill across flatter country started to rise so that, within a year or two, the banks seemed likely to be overtopped. The cause was found to be long bars of coarser sediment coming down the canal—presumably in slow motion from the cross-drainages. Urgent action was required so advantage was taken of a small cross-drainage well below canal bed, as shown in Fig. 13.1 (c). A “tunnel” was made with its roof at canal bed-level, one end was closed and the other was gated; slots were left in the roof. When a bar arrived the gate was raised and the extra load washed into the cross-drainage and thence into the adjacent river. Success was complete and the general design became standard. Plate 20 shows a later ejector of essentially the same design. The “slots” here are front openings, and there are divide walls inside the tunnel to provide some streamlining. The work was for the channel modelled in the left of Plate 18; the overall scheme was to divert gravel of the Upper Bari Doab canal into the abandoned inundation canal (the left channel) and then eject it (Plate 20) back to the river. An interesting feature of the design was that the tunnel top was built above bed-level under the impression that the “Pitot effect” would aid collection of gravel;
actually the bed rose locally to the tunnel top and the work then operated like the original Crump design with “slots” in the roof. Crump ejectors have been used extensively for sand beds with total load varying from bed sizes down. Here the efficiency has to be small, compared with gravel where total ejection is possible, so common practice is to build a sequence of two or three; in flat country the small drop in bed-factor justifies the cost.
CHAPTER 14

PHILOSOPHY

14.1. Introduction. The pure science of regime theory has grown on inductive lines, with its own set of phenomena, and does not depend on beliefs of conventional engineering fluid mechanics of the rigid boundary. For this reason alone the methods and related results of the two subjects deserve comparison. The applied science of regime theory has been developed in terms of applications of basic equations to progressively more complex situations to which they cannot apply rigorously, and the steps have been accompanied by discussions and verifications aimed at preventing the approximate and not-quite-true from being confused with the exact and "true". This development treats the reader as an equal in judging practical uses in terms of his own logic and, as it cannot be a practice in all texts, it too merits some discussion. Accordingly this chapter presents a variety of philosophical points for the consideration and criticism of the reader who is well versed in general fluid mechanics; the purely practical reader may omit mathematical references without loss.

14.2. Physical indications of regime formulas. The three dynamical statements made by basic regime formulas are already on record (para 7.3) and need no further comment. Attention is drawn, however, to the flow formula's alternative forms, equations (7.3) and (7.3b) with $C \to 0$. The first is of the "smooth boundary" type, in fact that of Blasius, and the latter of the "rough boundary" type. The former was derived algebraically from empirical results of King, 1943, and was unexpected. Once discovered it drew attention to the fact that a "smooth boundary" type might have been foreseen. The word "smooth", applied to a rigid boundary, means only that the resistance to flow can be expressed entirely in terms of the fluid properties and flow parameters because there is a thin layer of fluid in a special phase at the boundary and the shear stress in it can be so expressed; alternatively one could rephrase to suit energy dissipation rate instead of resistance. Now, the regime bed is also self-formed, but out of the water-sediment complex, and its properties are expressed in terms of $V^2/d$, $v$ and (if relevant) $C$; the sides are rigid but hydraulically smooth. Moreover no dif-
ference in general flow patterns, in rigid or mobile boundary hydraulics, has ever been noticed associated with boundary nature. Therefore the functional form of the resistance equation for any case ought to be a special form of that for the most general case which, in the present context, is the moving duned bed with smooth sides. Equation (7.3b) is of the “rough” boundary type, as should be expected from the corrugations that the dunes present, and is expressed in terms of an equivalent roughness height which, presumably, is some kind of measure of dune height and shape as determined by the nature of the water-sediment complex. This peculiarity of one and the same bed being smooth or rough according as it is viewed in terms of the complex that built it, or of the visible structure built from the complex, is partly responsible for the idea (para 14.5) of a functional form of the friction factor common to all types of boundary.

14.3. Potentiality of the Blasius Equation. The Blasius equation (Goldstein, 1952) for smooth pipes:

\[ f = \frac{0.316}{(Vd_f)^{1/4}} \]  

was found by plotting data on a friction factor diagram up to Reynolds Number of about 10^5. Regime theory data support the Blasius functional form up to about 10^9, which is far beyond anything reached in the laboratory but is in the range of large hydroelectric culverts. Figure 4.2 (from Blench, 1961, 1963) shows the Blasius and regime ranges. It shows also, as “N, smooth, extrapolation” the smooth boundary line of the well-known Moody Diagram; this line results from integrating the speculative logarithmic velocity distribution formula (para 14.11) that Prandtl, 1943, 1953, has consistently stated does not apply rigorously to pipes (and, for the same reason, canals). It is noteworthy that this curve does not agree well with the data of large hydroelectric culverts and it is apparently one of the set marked CW—for commercial metal pipes—that shows departure from “smoothness” at a high enough Reynolds Number. Levin, 1966, discusses its defects. The expectable failure of the logarithmic formula raises the question why the Blasius form should succeed. An explanation starts from the fact that there are moving-bed phases without dunes and the ordinary rough-boundary type of formula fits them. Apparently, then, the Blasius form is peculiar to a duned moveable boundary not to any moving one as para 14.2 suggests. Now, Prandtl and Tietjens, 1957, record “wavy” rigid boundaries fitted by the Blasius form, and Einstein, 1956, found periodic instabilities in the so-called
laminar film on a rigid boundary. Perhaps, therefore, the Blasius form pertains to hydraulic conditions that permit periodic instabilities to occur adjacent to the boundary; when a bed is mobile it forms waves to suit the periodicity; when it is “wavy” it just happens to have wave lengths consonant with the instabilities.

14.4. **The “universal” velocity distribution formula.** The remarkable family resemblance among velocity distribution curves, regardless of boundary nature or shear stress has produced, long ago, the idea that there should be some kind of “universal” (as it is called in literature) velocity distribution relation, rather than different ones for different boundaries and shears; more prosaically “universal” means non-dimensional. Prandtl, 1943, and Goldstein, 1952, attribute the presently accepted:

\[
\frac{(u_{\text{max}} - u)}{V_*} = f_n(y/d) \tag{14.2}
\]

to D’Arcy, about the middle of last century. Here \(y\) locates the point at which \(u\) is measured and \(d\) is a linear measure of sectional size. This formula is not intended to apply too close to the boundary, but some texts do not mention this qualification. Its applicability is to the flat portion of the curves in Fig. 14.1 (from Blench, 1961 (b), 1963); where the velocity dips violently to zero as the boundary is approached the Prandtl formula takes over. Attempts to deduce the form of the function in equation (14.2) have been mainly along the lines of idealized mathematical or dynamical “models”, with the aid of analogues to assist visualisation of turbulent transfer of momentum. The principal models are the momentum transfer, vorticity transfer and similarity ones described by Goldstein, 1952. They disagree a little with each other and with the rather scanty laboratory data cited to test them; the data disagree with each other also. Fig. 14.1 plots two sets to illustrate. Elementary engineering fluid mechanics texts confine themselves to the momentum transfer model, explained in terms of the Prandtl mixing length analogue; the analogue was obviously taken from the kinetic theory of gases and does not depict turbulence at all although it gives one means of visualizing transfer of momentum of one direction by particles moving at right angles to that direction. The texts also confine attention to the data of Nikuradse, 1933, alone. Statistical theory (Goldstein, 1952, Sutton, 1955, Hinze, 1959) is removing the need to use analogues for argument. A factual scientific treatment is by Dryden and von Karman, 1956.

14.5. **The “universal” flow formula.** Obviously, if the final universal velocity distribution formula was known, it could be integrated
to obtain an expression for mean velocity, and this would be a
universal flow formula. Actually the expression for the friction
factor:

\[ f = \frac{2gdS}{V^2} = fn(Vd/\nu, e/d, X, \text{shape}) \]  \hspace{1cm} (10.10, 14.3)

is the latter formula in general terms, and is of no quantitative
use till the form of the function is known. In trying to find
dynamical statements behind the original Lacey equations the
author (Blench, 1938) found himself forced, naturally, to think
in terms of a universal flow formula. There seemed to be an
incongruity in the situation that the flows with similar-
looking rough, transitional and smooth rigid boundaries and
the appreciably different mobile duned one, seemed essentially the
same; yet the smooth one had a flow formula of quite different ap-
pearance than the rough boundary type that suited both rough rigid
and mobile-duned. It seemed that, regarding a flow formula as a
resistance one, the boundary, whatever its phase or appearance,
was merely a “brake” whose “brand” (smooth, rough, etc.) was
inconsequential to the flow pattern unless interest centred on be-
aviour practically in contact with the “brake”. But if this were
so, then it seemed that the dependence of \( f \) on \( e/d \) alone for rough
rigid boundary meant that the “brake” can be measured generally
in terms of an equivalent roughness height, and therefore the
Reynolds Number in the Blasius smooth boundary formula mea-
sures a relative smoothness. Therefore the comparison of em-
pirical formulas of different boundary types might disclose a
common form in terms of an equivalent roughness height.

14.6. Although the King equation, which generalizes Blasius, was
not known at the time, the form:

\[ 1/\sqrt{f} \alpha (d/x)^{1/4} \]  \hspace{1cm} (14.3)

was suggested immediately by the preceding viewpoint. Then:
i. \( x = f_L (\nu g)^{2/3}/g^2 \) gives Lacey’s form \( V \alpha f^{-1/4}R^{3/4}S^{1/2} \)
ii. \( x = \delta \), a “laminar film thickness”, gives Blasius’ \( f \alpha (Vd/\nu)^{-1/4} \)
provided \( \delta \) is defined by \( d/\delta \alpha (Vd/\nu)^{1/2} \)
iii. \( x = e \), a roughness height, gives a slightly amended Man-
ning’s equation with the index 2/3 replaced by 3/4.

In terms of present regime equations:
iv. \( x_{a}(\nu F_S)^{1/2}/F_b(1+C/233)^2 \) gives equation (7.3b)
v. \( x = \delta' \), a boundary zone thickness, gives equation (7.3a)
   provided \( b/\delta' \) is defined to be \( (1+C/233)^2 (Vb/\nu)^{1/2} \), and
   \( b \) is taken to be the “representative size” denoted by \( d \) in
equation (14.2).
14.7. **Meaning of a “laminar film thickness”**. The rather arbitrary definition of $\delta$ in the preceding paragraph is in accordance with normal fluid mechanics practice. From a purely logical viewpoint one should not use a definite expression for such a quantity. All that is known is that there can be a very thin zone, adjacent to a visibly “smooth” boundary to generally turbulent flow, where the time-mean velocity distribution seems to be of the parabolic form that occurs in laminar flow. Stanton, 1922, demonstrated the existence. There is no definite division line between this zone and the adjacent turbulent one. However, the existence of the laminar zone explains a lot about behaviour in general and one cannot avoid thinking of it; therefore some kind of expression that gives it at least an order of magnitude is desirable. In rigid boundary fluid mechanics it has become conventional to find where the accepted turbulent zone curve, that does not apply in a transition between itself and the laminar film, meets the tangent to the laminar film velocity curve. The distance of this intersection from the boundary (assumed geometrically smooth) is the “laminar film thickness”.

14.8. It is of philosophical interest to consider how a relation like $(d/\delta) \approx (Vd/\nu)^{1/2}$ might arise from a typical “plausible assumption” argument of the kind common in rigid-boundary fluid mechanics. Using the Navier Stokes equation with the rate of flux of turbulent momentum terms, very close to a canal boundary, once in the turbulent zone and once in the laminar one, we have, with conventional notation:

\[
\frac{\partial}{\partial y}(\rho u'v') = gS \text{ near the bed, with } Oy \text{ measured downwards.}
\]

\[
gS = -\nu \cdot \frac{d^2u}{dy^2} \text{ at the bed}
\]

\[
\therefore \frac{\partial}{\partial y}(\rho u'v') = gS \text{ near the bed, with } Oy \text{ measured downwards.}
\]

\[
gS = -\nu \cdot \frac{d^2u}{dy^2} \text{ at the bed}
\]

\[
\therefore \frac{\partial}{\partial y}(\rho u'v') = gS \text{ near the bed, with } Oy \text{ measured downwards.}
\]

\[
\delta \approx c \cdot (Vd/\nu)^{1/2}
\]

14.9. **Postulate of universal flow formula.** In terms of the preceding discussion there is merit in making a postulate, for use in seeking flow formulas for new cases (e.g. for phases to which the regime formulas do not apply and perhaps for non-Newtonian flow) and for coordination with rigid boundary formulas of various empirical or speculative types. It is:

“The friction factor for any type of boundary or fluid depends on, and is probably proportional to, the square root of an equivalent relative roughness, for any given conduit shape. The equivalent roughness height will be expressible in terms of suitable parameters of the boundary, the flow and the fluid”.

148
14.10. Comparison between regime theory development and boundary layer theory. In coordinated works on boundary layer theory and related topics, at top technical level, (Schlichting, 1960, Prandtl, 1953, Goldstein, 1952) there is a base of pure hydrodynamics, a rigorous use of the Navier-Stokes equation for the few soluble laminar flow problems and a couple of applications to turbulence. Thereafter the need to solve practical problems causes deviations from rigour and various stratagems are used to obtain practically useful answers. But every deviation from rigour is admitted and explained, and there is never any excuse for mistaking the non-rigorous for the basic. The development in the present text has tried to follow this policy although, of course, from an inductive base. It is impracticable for texts on elementary engineering mechanics to explain how practical applications deviate from basic theory that is not taught fully at an engineering level; therefore the engineering reader is wise to assume that all material not readily understood may be non-rigorous and, perhaps, dynamically incorrect in the absolute sense.

14.11. Logarithmic flow formulas. In view of the postulate in para 14.9 the ingenious indirect arguments leading to the well-known logarithmic flow formula of engineering fluid mechanics texts, and thence to the well-known Moody Diagram, are interesting. The starting point was the Prandtl attempt to find velocity distribution in the wind blowing along the ground, summarized by him recently (Prandtl, 1953, Sec. III, 5). The case was reduced to virtually an infinitely deep smooth canal. The argument was, basically, that the rate of turbulent momentum transfer per unit area, \( \rho u'v' \), far enough from the bed for viscosity to have no effect, would depend on \( \rho \), on \( du/dy \) and presumably on nothing else but position \( y \) above the ground; if so it would have to be proportional to \( \rho y^2 (du/dy) \). However, the argument was illustrated from the mixing length analogue. This gave the classic:

\[
u/V_x = (1/k) \cdot \log_y/C \quad \text{(14.5)}\]

Obviously this should not be used for a pipe, or canal, that calls for inclusion of a size \( d \) in the argument, and Prandtl, 1943, 1953, was always careful to say so. However, the formula can be adapted quite well to pipes and canals as an empirical fit. The problem of how to use it to fit the zone near the ground where viscosity enters was disposed of by putting \( C \), which must have the dimensions of length, proportional to \( \nu/V \star \) as this was the only
length that could be invented out of $v$. Thence to suit a “rough” ground $C$ was replaced by roughness height, $e$; this seems reasonable enough in terms of the behaviour of $f$ in the friction factor diagram. Engineering texts integrate the formula over a pipe to obtain mean velocity $V$ and, therefore, a flow formula which gives $f$. This form of $f$ for smooth boundary gives the lowest line in the Moody Diagram and, for rough boundary, defines the scale of $e$ on the same diagram; the constants come from Nikuradse’s, 1933, experiments. Texts usually omit Prandtl’s warnings about lack of rigour; Dryden and von Karman show how necessary they are.

14.12. **Velocity distributions by dimensional analysis.** Dimensional analysis brings into focus the imperfections of various “models” of velocity distribution and indicates that, with sufficient ingenuity, a variety of formulas could be “proved” by suitable “plausible assumptions”. To illustrate from a broad flat-bottomed canal of uniform slope and infinite length, in steady flow, one may specify the problem as “If a steady discharge intensity $q$ is maintained in a canal that is tilted so that the body force per unit mass on the fluid, resolved in the direction of flow, is $gS$, if the fluid has properties $\rho$, $\nu$, and if the boundary nature can be described by a roughness height, $e$, then the motion is determined. Therefore, using conventional symbols:

$$\frac{\tau_{0}/\rho, d}{u, u'v', d^n u/dy^n} = \text{fn}(q, gS, \nu, e)$$

$$\text{fns}(q, gS, \nu, e, y)$$

where $y$ specifies, in some way, the position of the point functions on the left of the relevant equation”.

14.13. The preceding relations will be used for the zone in which velocity distribution appears to be unaffected by viscosity; this zone is the approximately 85% of depth measured from the free surface (Fig. 14.1) where the small velocity gradient justifies neglecting the viscous shear stress, $\mu du/dy$, in the Navier-Stokes equation:

$$\rho u'v' = \rho gyS + \mu \frac{du}{dy} \quad (14.6)$$

Here $u'v'/gyS = \left(\frac{u'v'/V_z}{gS}\right)(d/y)$ is approximately unity. $y$ is measured from the surface down.

14.14. **To prove a parabolic velocity distribution.** From the relations of para 14.12 it is permissible to state that:

$$\text{fn}(d^n u/dy^n, d, gS = V_z, \nu, e, y) = 0$$

which can be organized to:

150
Physically, from knowledge of the friction-factor diagram, one should expect \( V_\ast \frac{d}{d\nu} \) to concern only whether flow is turbulent or not; \( V_\ast \frac{y}{\nu} \) should be expected to be relevant for small enough values of \( y \) (if \( y \) is measured from the boundary) but we have agreed to confine attention outside this zone; \( e/d \) can hardly be relevant if the boundary is "smooth" and it probably does not matter except near the boundary anyway. So, it seems that beyond some distance from the boundary:

\[
\left( \frac{d^2}{V_\ast} \right) . \left( \frac{d^3 u}{dy^2} \right) = \text{constant}
\]

whose solution is:

\[
\left( u_{\text{max}} - u \right) /V_\ast = A (y/d)^{3/2} \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad (14.7)
\]

if, for convenience, we measure \( y \) from the surface down and make \( du/\nu = 0 \) at \( y = 0 \) as it must be, physically.

14.15. To prove a semi-cubic parabolic distribution. Here the basic form is chosen as:

\[
\text{fn}(du/\nu, d, gyS, \nu, e, \sqrt{u'v'}) = 0
\]

whence:

\[
\text{fn}[ (du/\nu)^2 . d^2/\sqrt{u'v'}, (\sqrt{u'v'}) /gyS, \sqrt{gyS} . d/\nu, e/d] = 0
\]

We reject the second term as practically constant by equation (14.6), the third as irrelevant to the 85% zone because it contains viscosity, and the last for the same reason as in para 14.14. The first term is then accepted as practically constant and:

\[
\left( u_{\text{max}} - u \right) /V_\ast = A (y/d)^{3/2} \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad (14.8)
\]

because \( u'v' = V_\ast ^2 (y/d) \)

14.16. To prove the von Karman log-surd distribution. Write:

\[
\text{fn}(du/\nu, d^2 u/\nu^2, \sqrt{u'v'}, gyS, \nu, \nu) = 0
\]

whence:

\[
\text{fn}([du/\nu)^4 . (d^2 u/\nu^2)^{-2} . (gyS)^{-1}, \sqrt{u'v'}/gyS, gyS/(\nu du/\nu),
\]

\[
(\sqrt{u'v'}) e^2 /\nu^2] = 0
\]

and, as usual, reject terms in \( \nu \) and \( e \), and reject \( \sqrt{u'v'}/gyS \) as practically unity if \( y \) is measured from the surface. Then the von Karman differential equation:

\[
(du/\nu)^2/d^2 u/\nu^2 a(gyS)^{1/2}
\]
remains. Its solution is given in engineering texts as:

\[ (u_{\text{max}} - u) / V_* = A \log_e (1 - \sqrt{y/d}) + \sqrt{y/d} \] .... (14.9)

which has a physically impossible small finite velocity gradient at the free surface because zero gradient there would compel an infinite integration constant and, therefore, an impractical solution. Eq. 14.9 is really half a solution meeting its mirror image in a cusp on the centerline; so \( d^2u/dy^2 \) is infinite there and the differential equation is satisfied at \( y=0 \) despite \( du/dy \) not being zero.

14.17. To prove the Prandtl log distribution. Write:

\[ f_n (du/dy, V_*, d, y, v, e) = 0 \]

and reorganise to:

\[ f_n [(du/dy), (y/V_*), V_*y/v, V_*d/v, ejd] = 0 \]

As \( u'v' \) is not used there is no need to restrict \( y \) to being measured from the surface, so suppose it measured from the boundary. Arguments as before make only the first term likely to matter much beyond some distance from the boundary. So there:

\[ du/V_* = dy/y = d(V_*y/v)/V_*y/v \]

whence:

\[ u/V_* = A \log_e V_*y/v + B \] .................................................. (14.10)

This, like the von Karman formula, has to give a cusp on the centre-line and negative infinite velocity at the boundary.

14.18. A conclusion from the preceding analysis seems to be that if, with sufficient dynamical insight, a non-dimensional, or “universal”, velocity distribution formula were discovered, it would contain secondary terms that present somewhat speculative formulas omit. As there is no obvious way of discovering it, at present, there is merit in concentrating on the simple universal flow formula line of thought (para 14.5) which might even give some clues to new guesses at velocity distributions.

14.19. Meyer-Peter type of boundary activation formula. The universal flow formula postulate suggests a slight amendment to the useful Meyer-Peter, 1948, formula for the condition at which a loose stone boundary would just start to move. His formula is:

\[ q^{2/3}S = 0.25D \] ................................................................. (14.11)

where the units are now feet and seconds, and \( D \) is stone size. One might argue that movement will occur when tractive force on the area occupied by a stone equals a coefficient of friction
multiplied by the buoyant weight of the stone. If so:
\[ \tau D^2 \propto D^3 \]
But in a broad channel \( \tau \propto dS \), so:
\[ dS/D = \text{const.} \]
If we accept that \( V = \text{const.} \) \((d/D)^{1/4} \sqrt{gdS}\) and write \( V = q/d \), then elimination of \( d \) from the preceding equations leads to:
\[ q^{2/3}S^{5/6} = 0.5D \]  \hfill (14.12)
using a coefficient 0.5 to give reasonable consistence with equation (14.11).

14.20. **Aspects of** \( V^2/d \). In speculating on physical meanings that might be indicated by the utility of the bed-factor, in a certain phase, the possibility arises that it is a principal factor representing the ratio of the lift per unit mass of bed-material to the submerged weight; the implication is that the bed-material has to be lifted before it can move. If this is correct then \( V \) times \( V/d \) is a measure of lift, and the bed-factor becomes, for more general work, the “densimetric Froude Number” \( V^2/((s-1)gd) \) where \( s \) is specific gravity of solids. However, A.S. Qureshi, 1965, introduced this densimetric number into analysis of bridge pier scour experiments by arguing that the criterion should be the ratio of particle drag to particle submerged weight regarded as a co-efficient of friction; this compelled him to introduce a drag coefficient (necessarily idealized) into the numerator. From the viewpoint of model scaling \( V^2/gd \) may be regarded as \((d_c/d)^3\) where \( d_c \) is critical depth; then a fixed bed-factor implies a fixed ratio of regime depth to critical.

14.21. **Aspects of** \( Vb/\nu \). Surprise is often expressed at the presence of a Reynolds Number in terms of breadth in regime equations. It is merely \( \mu \rho V^2/b \) made non-dimensional conventionally by dividing by \( \rho^2V^4 \). From this viewpoint equation (7.3) may state that shear stress cannot suddenly change from bed value to side value. Historically \( V^3/b \) came to notice by Blench (1941) estimating, on conventional rigid-boundary lines, a principal factor in shear stress, assuming canal sides were hydraulically smooth. Once the result was obtained it became obvious that Lacey’s \( p = 7.11Vr \) was \((V^2/r)/(V^3/p) = 7.11\) and merely stated that he had averaged out the “relative importance” of sides and bed in the data he had analysed. His formula, in its more common form \( p = 2.67Q^{1/2} \), has received continuous strong verification as to the index, 1/2 over a very large range of \( Q \), but the coefficient has been shown repeatedly to be different in different systems (e.g. Fig. 3.2). Apparently
the formula is physically meaningful, and this makes \( V^3 / p \) as meaningful as \( V^2 / r \) which is effectively a Froude Number.

14.22. **Aspects of the Vig Number, \( \sqrt{\nu \text{r} \cdot \text{D}} / \nu \)** This particle Reynolds Number (paras 7.16, 10.11), in terms of a velocity \( (\nu \text{g})^{1/3} \) which may be called “Vig” (from viscosity-gravity), was introduced during dimensional analysis by Blench and Erb, 1957, because it met the requirement (para 7.3(i)) that the Froude Number in terms of depth had to depend on water-sediment complex properties only. Vig, although not commonly known or used, appears naturally in the simple ship-model problem of text-books when calculating the viscosity scale for dynamical similarity. In fact, if \( V^2 / gL \) and \( V L / \nu \) are independently constant, so is their product \( V^3 / \nu \text{g} = (V / \text{Vig})^3 \); therefore the viscosity scale must be the cube of the velocity scale which is the 3/2 power of the length scale. Obviously, the drag coefficient could be expressed as a function of the Froude Number and the Vig Number, or the Reynolds Number and the Vig Number. Thus, Vig seems appropriate to situations where gravity waves occur in the presence of viscosity. Lamb, 1959, para 348, illustrates a problem where the least wind that could maintain waves against viscous damping is proportional to Vig in which gravity has been increased in the ratio of fluid to air densities. Keulegan, 1949, deduces a “criterion of mixing” in stratified flow. The criterion is that at which instability occurs in interfacial waves and is obtained by conventional hydrodynamics; it is proportional to Vig divided by the relative flow velocity of the strata, but with gravity altered in the ratio of density difference to density of the heavy layer. Lacey 1929-30 corrected his \( p / r \) proportional to \( V \) relation (para 14.21) by dividing \( V \) by Vig. An interesting relation is obtained by writing the standard equation for fall velocity, \( U \), of a particle of fixed shape in still fluid. The result is:—

\[
C_d \text{a} (s -1) (UD/\nu)^{-2} (Vig D/\nu)^3
\]

where \( C_d \text{a} \) is the drag coefficient. Finally, the Vig Number is the algebraic equivalent of \( (Vig/V) \) \( (Vd/\nu) \) \( (D/d) \) and of \( (VD/\nu)^{2/3} \) \( (gD/V^2)^{1/3} \).

14.23. **Novel viewpoints on sediment transport mechanics.** The complementary natures of regime theory and conventional sediment mechanics (para 6.3) requires that users of the former keep advances of the latter in view. The following, that do not appear in ASCE, Sept. 1963, deserve mention.

14.24. Bagnold, (1954, 56) performed viscometer experiments on
suspensions of particles of the same specific gravity as the fluid and measured the direct stress exerted by the particles on each other. He produced a simple theory of the interaction of transported particles and was led, thereby, to plot certain parameters of the experiments against each other. The plots indicated that, when inertia effects predominated, shear stress could be represented as proportional to the product of velocity gradient squared and the square of a kind of “mixing length” proportional to the product of particle size and a linear dispersion measure; when viscous effects predominated the shear stress was proportional to a velocity gradient times the fluid viscosity times a power of the linear dispersion factor. He used the idea of an internal friction coefficient related to particle impact.

14.25. Duplessis, 1965, used Gamma rays to measure sediment concentration in pipes near the “fall-out” condition and found some support for the Bagnold friction coefficient idea. He found also that the exponential type of rule for suspended load, used for small suspensions, applies reasonably well (considering that circular pipes were used) near “fall-out” and graded continuously into the bed concentration, which is that for static sand. His thinking was influenced strongly by Bagnold’s ideas.

14.26. Langbein, 1964, proposed that the indices in self-adjustment (including transport) equations could be deduced from certain minimizing conditions of statistical type, and demonstrated how certain equations could be deduced. Criticism, in terms of the formal statistics and thermodynamics of the subject, was severe. However, the author sees the work as an appreciation of the minimal principles behind dynamics. The motivation parallels that which inspires physicists to convert Newton’s second law into Hamilton’s principle, or to a statement that a free particle describes a geodesic in space-time; the layman is similarly inspired when he says “Nature takes the line of least resistance”. As a universal principle cannot be deduced yet, a serious attempt to arrange formulas, purporting to represent real happenings, to test whether they disclose a suspected statement of principle deserves careful consideration. The arguments can be developed later if the principle becomes apparent; if they could be stated perfectly in advance the principle could be deduced.
REFERENCES


BLENCH, T., "River Hydrology, or Fluviology, For Engineers," Civil Engineering, Nov., 1966(b).


BLENCH, T., discussion of "Hydraulic Resistance of Alluvial Streams," ibid (b).


BLENCH, T., "Dimensional Analysis and Dynamical Similarity for Hydraulic Engineers," University of Alberta Bookstore, Edmonton, Canada, 1968(a).


BLENCH, T., "Coordination in mobile-bed hydraulics." Journal of the Hydraulics Division, ASCE. November 1969. (Condensation for IUTAM available from author.)


CALIFORNIA DEPT. PUBLIC WORKS. "Bank and shore protection in California highway practice." Nov. 1960. (From State Printing Division, North 7th St., Sacramento 14, Calif.)


FRIEDKIN, J. F., "A laboratory study of the meandering of alluvial rivers." U.S. Waterways Experiment Station, Vicksburg, Miss., 1945.


GILBERT, G. K., "The transportation of debris by running water." Based on experiments made with the assistance of E. C. Murphy, U.S. Geological Survey Paper 86. 1914.


INTERNATIONAL ASSOCIATION HYDRAULIC RESEARCH. *Proceedings*, March 1951. All papers on sediment exclusion or ejection.


JESSON, A. W., “History of channels of Lower Gugera Branch circa 1935.” To be maintained to date by Lower Chenab Canal Circle, Lyallpur, Punjab, West Pakistan.


KONDRATEV, N. E., “River flow and river channel formation.” Hydromet. Service of Council of Ministers of USSR. 1959. (Fig. 47.) Translation OTS, U.S. Department of Commerce.


LACEY, G., “Regime diagrams for the design of canals and distributaries.” Technical Paper U.P. Public Works Department, India (Irrigation Branch), No. 1. 1932.


161


MILLER, J. P., “High mountain streams; effects of geology on channel characteristics and bed material.” New Mexico Institution of Mining and Technology. Socorro, New Mexico. 1958.


NING CHIEN, Comments by Lacey, Lane and Blench on “A concept of the Regime Theory.” Translation, American Society of Civil Engineers, Vol. 122, 1957, p. 785.


GLOSSARY OF STANDARD TERMS
(Uncommon usage will be mentioned in proper paragraph; standard units are foot, slug, and second)

a A constant used in the regime slope equation
b Breadth of channel. For trapezoidal channel = cross-sectional area of flow divided by the depth d
c C A constant used in the regime slope equation
Cc Breadth of channel. For trapezoidal channel = cross-sectional area of flow divided by the depth d
cusec. Cubic foot per second

C Bed-load charge, measured as dry weight per second of bed-load divided by weight of water flow per second, and reduced to parts per hundred thousand
Cc Bed-load charge at critical velocity
d Depth of channel. For trapezoidal channel = distance from water surface to horizontal bed
D Length specifying average particle size of a natural bed material

\[ m_D \] Median D by weight
\[ w_D \] Median D by number
\[ n_D \] Particle size, defined according to context
\[ f \] The Lacey silt factor and the friction factor according to context.
\[ f_n \] An unspecified function of . . .
\[ F_b \] The bed factor \( V^2/d \)
\[ F_s \] The side factor \( V^3/b \)
g Acceleration due to gravity
K 3.63g/\( v^{1/3} \), from regime slope equation
M_h Meander breadth
M_l Meander length
p Wetter perimeter. Also bed-load in lbs./ft-sec.
Q Discharge rate in cusecs
q Discharge intensity = discharge per unit breadth
r Hydraulic radius = cross-sectional area divided by p
S Channel slope in feet per foot measured along flow
V Mean velocity at a section
\( V_s \) Terminal velocity of settlement of particle in the still fluid

\[ V_s = \sqrt{gdS} \]
\[ \nu \] Kinematic viscosity
\[ \mu \] Dynamic viscosity or coefficient of friction, depending on context
\[ \rho_f, \rho_t \] Mass density of fluid
\[ \rho_s \] Mass density of sediment particles

165
APPENDIX 1

USEFUL DATA

1 cusec = 1 cubic foot per second
   = 374 imperial gallons per minute
   = 449 U.S. gallons per minute
   = 0.0283 cubic metres per second

1 cusec hour = 1 acre inch
1 cusec day = 2 acre feet
               = 0.538 × 10^6 imperial gallons
               = 0.646 × 10^6 U.S. gallons

1 cusec year = 730 acre feet
1 inch = 53.3 acre-ft per sq. mile.
1 cusec day of water weighs 2,700 short tons

At C=1.0, 1 cusec day of water carries 0.027 short tons bed load
which, at 90 lb./ft.³ dry weight, would deposit in a space of 0.6 ft.³

At C=1.0, 1 cusec year of water carries 9.86 short tons bed load
which, at 90 lb./ft.³ dry weight, would deposit in a space of 219 ft.³
   = 8.1 yd.³

The weight of a sphere of s.g. 2.65 is 86D³ lb.
The buoyant weight of this sphere is 54D⁶ lb. in water.

1 short ton = 2,000 lb. = 0.907 metric tons
1 short mile = 5,000 ft. = 1.524 kilometres
1 foot = 305 millimetres.

Manning's n, by a rough formula, is Dᵢ¹⁶/₄₁ for unmoving gravel
or rock, where Dᵢ is the rock size in inches.

10⁻⁵ν = (70/T)₀.₈⁹ from T = 40° to 100°F

W lb. stone; s.g. 2.65; max. length L = 0.4W¹/³

Well-placed double layer, length horizontal, on bank slope A.
Normal T = 1.5L sinA.
## APPENDIX 2

### SUMMARY OF FORMULAS

**A. REGIME THEORY**

<table>
<thead>
<tr>
<th>BASIC</th>
<th>EQUATION</th>
<th>PARAGRAPH</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V^2/d = F_b )</td>
<td>(7.1)</td>
<td>7.6</td>
</tr>
<tr>
<td>( V^3/b = F_s )</td>
<td>(7.2)</td>
<td>7.6</td>
</tr>
<tr>
<td>( V^2/gdS = 3.63(Vb/v)^{1/4} )</td>
<td>(7.3)</td>
<td>7.6</td>
</tr>
<tr>
<td>( gVS = F_b \cdot (rF_s)^{1/4}/3.63 )</td>
<td>(7.4)</td>
<td>7.7</td>
</tr>
</tbody>
</table>

**NEAR BASIC**

\( V^2/gdS = 3.63 \left(1 + C/233\right) \left(Vb/v\right)^{1/4} \) \( (7.3a) \)

\( V = (d/x)^{1/4} \left(gdS\right)^{1/2} \) \( (7.3b) \)

where the equivalent roughness height is:

\( x = \left(rF_s\right)^{1/2}/\left[3.63^2 F_b \left(1 + C/233\right)^2\right] \) \( (7.3c) \)

\( gVS = F_b \cdot \left(rF_s\right)^{1/4}/\left[3.63 \left(1 + C/233\right)\right] \) \( (7.4a) \)

**BED AND SIDE FACTORS**

- **Subcritical.** \( F_b = F_{bo} \left(1 + 0.12C\right) \) \( (7.5) \)

- **Sand.** \( F_{bo} = 1.9\sqrt{D_{mm}} \) \( (7.6) \)

- **General.** \( F_{bo} = 0.58 V_{s}^{11/24} \left(v_{70}/v\right)^{11/72} \) \( (7.6a) \)

- **Gravel.** \( F_{bo} = 7.3 \sqrt{D_{f}^{1/4} \left(v_{70}/v\right)^{1/6}} \) \( (10.12) \)

Rules for side-factor in Secs. 7.18 and 10.52.

All formulas and rules to be read in context, with attention to phase.

**DERIVED FOR DESIGN & ANALYSIS**

- \( b = \sqrt{F_b Q/F_s} \) \( (7.8) \)

- \( d = 3\sqrt{F_b Q/F_s^2} \) \( (7.9) \)

- \( S = \left[F_{bo}^{11/2} F_s^{1/12} / KQ^{1/6} \right] \cdot f'(C) \) \( (7.11') \)

- \( S = \left[F_{bo}^{7/8} / K^{1/4} d^{1/8} \right] \cdot f''(C) \) \( (7.11'') \)

- \( S = \left[F_{bo}^{11/12} / K^{1/6} Q^{1/12} \right] \cdot f'''(C) \) \( (7.11''') \)

where \( K = 3.63 \ g/v^{1/4} \)

and the functions of \( C \) are given by Fig. 7.2.

- \( d = (q^2/F_b)^{1/3} \) \( (7.12) \)

- \( V = (F_b F_s Q)^{1/6} \) \( (7.13) \)

**RIVERS**

\( M_L = 10b \) \( (3.1, 10.22) \)

Apply meander correction coefficient \( k \) to all slope formulas \( 10.36, 11.2 \)

For scour see \( 11.39, 40 \)
B. KELLERHALS PHASE (paras 10.47, 48)

\[ gdS \propto D^{1+n} \]  
\[ V^2/gdS \propto (d/D)^{1/2} \]  
\[ F_{ho} \propto (D/d)^{1/2} \cdot D^n \]  
\[ d \propto D^{-(1+2n)/5} q^{4/5} \]  
\[ S \propto D^{(6+7n)/5} q^{-1/5} \]  
\[ V \propto D^{(1+2n)/5} q^{1/5} \]

For \( C \to 0 \)
PLATE 1(a). Model sand river; 0.1 cfs steady; sinusoidal; chute development. Low water exposes pattern. Courtesy U.S. Waterways Experimental Station.

PLATE 1(b). Sinusoidal meander in gravel outwash from glacier. Courtesy Geol. Surv., Canada; photo H. S. Bostock.
PLATE 5. Detail from Plate 4. Flood shows ancient meanders. Courtesy Govt. of Alberta.
PLATE 7. Artificial Pembina cut-offs, upper new; lower 5 years. Courtesy Govt. of Alberta.
PLATE 10. Ice jam, McLeod R., Alberta. Courtesy Govt. of Alberta.
PLATE 11. Washita River, Oklahoma. Sand bed; 1% total load. Rapid delta formation in power reservoir. Photo length about 7 miles. Courtesy Mr. Reuel W. Little, Attorney, Madill, Oklahoma.


PLATE 17(b). Model for Sukkur Barrage, Pakistan, applying principle of Plate 17(a) to canal offtakes. *Proc. IAHR*. 1951, p. 229
FIGURE 2.1. Typical cross-section of river valley, looking downstream. Vertical scale exaggerated 10 times, see para. 2.15. Courtesy C. R. Neil 1965 (a).
FIGURE 3.1. Relation of meander length to channel breadth.
FIGURE 3.3. Regime slope analysis chart (Blench and Qureshi, 1964).
FIGURE 3.4. "Log normal" bed-sand grain distribution from Fraser River, B.C.
FIGURE 4.2. Mobile and rigid boundary friction factor data.
FIGURE 5.1. Berm formation in channels.
FIGURE 5.2. Typical observed Punjab canal section; depths in feet.
FIGURE 71. Relation of nominal diameter and fall velocity for naturally worn quartz particles. Courtesy Interagency Committee on Water Resources, 1957.
$f'(C) = \frac{(1 + 0.12C)^{5/6}}{1 + C/233}$

$f''(C) = \frac{(1 + 0.12C)^{7/8}}{1 + C/233}$

$f'''(C) = \frac{(1 + 0.12C)^{11/12}}{1 + C/233}$

NOTES

(1) Duned beds; para 7.22
(2) Doubtful for $C > 10$
(3) --- Cooper and Peterson trend, para 10.27
(4) Intended for log-normal material

FIGURE 7.2. Bed-Load Charge Functions.
1. Empirical, approximate, practical. For regime theory phase.
2. For sediment specific gravity about 2.7 and ordinary stone shapes.
3. If $F_{bo} < 38 (d/D)^{-1/2}$ see Fig. 10.4.
4. Sediment diameter may be spherical, intermediate, sieve, or settlement.
5. Devised initially for regime slope analysis of rivers.

**FIGURE 7.3.** Chart for Estimating $F_{bo}$. 

MEDIAN DIAMETER BY WEIGHT, D - (ft.)

$F_{bo}$ - (cm./sec.)

$F_{bo} = 0.58 V_s^{1/24}$

$F_{bo} \sim D$
FIGURE 9.1. Bed-load formulas and river observations. (Blench 1964a, b).
FIGURE 10.1. Bed-factor analysis of subcritical Gilbert data. [Blench 1955 (b)]
FIGURE 10.2. Bed-factor analysis of supercritical sheet flow Gilbert data. [Blench 1955 (b)]
FIGURE 10.3. Analysis of King's Number from Gilbert data. [Blench 1955 (b)]

\[ C'_{\text{OBS.}} = 3.63 \left(1 + \frac{C}{400}\right) \]
\[ C'_{\text{CALC.}} = \left(\frac{V^2}{gdS}\right) + \left(\frac{Vb}{\nu}\right)^{1/4} \]
\[ F_{bo} = 38(D/d)^{1/2} \]

FIGURE 10.4. Sub-critical Phases of \( F_{bo} \).
FIGURE 10.5. Individual Experimenters' Bands (Hoque).
FIGURE 10.7. Visible Phases on Slope Chart.

<table>
<thead>
<tr>
<th>NO.</th>
<th>MIN.</th>
<th>MAX.</th>
<th>MEAN</th>
<th>PHASE</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>100.0</td>
<td>250.0</td>
<td>183.8</td>
<td>Plane bed</td>
</tr>
<tr>
<td>b</td>
<td>250.0</td>
<td>500.0</td>
<td>335.6</td>
<td>Ripples</td>
</tr>
<tr>
<td>c</td>
<td>500.0</td>
<td>1000.0</td>
<td>466.3</td>
<td>Dunes</td>
</tr>
<tr>
<td>d</td>
<td>600.0</td>
<td>1000.0</td>
<td>758.0</td>
<td>Transition and Sheet Flow</td>
</tr>
<tr>
<td>e</td>
<td>1000.0</td>
<td>2500.0</td>
<td>1285.6</td>
<td>Stnl. Wave, Antidune &amp; Chute Pool</td>
</tr>
</tbody>
</table>
FIGURE 10.8. Phase Loci (Simons and Richardson).
FIGURE 10.9. Roughness at one D (Simons and Richardson)
WEAK BOIL

A. TYPICAL RIPPLE PATTERN

B. DUNES AND SUPERPOSED RIPPLES

C. DUNES

D. WASHED-OUT DUNES OR Transition

E. PLANE BED

F. ANTIDUNE STANDING WAVES

G. ANTIDUNE BREAKING WAVE

H. CHUTE AND POOL

FIGURE 10.10. Diagram of bed-form types (Simons and Richardson).
FIGURE 11.1 Training works at barrages.
FIGURE 11.2. Apron designed for settlement to form protected bank to base of scour hole.
$W = \frac{0.00002 V^6 s_gR \csc^3(\rho - \alpha)}{(s_gR - 1)^3}$

$W =$ WEIGHT OF CRITICAL STONE IN POUNDS, TWO THIRDS OF STONE SHOULD BE HEAVIER

$\rho =$ $70^\circ$ CONSTANT FOR BROKEN ROCK

FIGURE 11.4. Rip-Rap design criteria compared.
FIGURE 12.1. Model breadth and depth scales compared. [Blench 1951 (a)].

N.B. \( d = \left( \frac{F_s}{F_b} \right)^{2/3} b^{2/3} \)
\begin{equation}
\frac{u}{V_e} = \sqrt{\frac{8}{\pi} \int_0^R \left( \frac{fn(r/R)}{r} \right) dr}
\end{equation}

where $B = 2/R^2 \int_0^R fn(r/R) dr$

\[ fn(r/R) = \begin{cases} 
6.56(r/R)^2 & \text{if } r/R \leq 0.13 \\
13 \left[ \cosh (r/R) - 1 \right] & \text{if } 0.13 < r/R \leq 1 \\
-2.5 \log_b (1-r/R) & \text{if } r/R > 1
\end{cases} \]

From 6 to 8 Nikuradse sets for $R/e = 507$

Sample from Stanton

FIGURE 14.1. Velocity deficiency formulae and some Nikursade data.